IIT – JEE: 2017 MATHS ADVANCED ASSIGNMENT DATE: 08-04-2017

TOPIC: TRIGONOMETRY

Single Choice Questions

1. Two equilateral triangles are constructed from a line segment of length L. If M and m are the maximum and minimum value of the sum of the areas of two plane figures, then

(A) M = 2m (B) M = $\sqrt{3}$ m (C) 2M = $3\sqrt{3}$ m (D) M = 4m

2. α, β and γ are the angles of triangle. If $\sin\alpha + \sin\beta = \sin\gamma\left(\cos\alpha + \cos\beta\right)$ then γ is equal to

(A) $\sec^{-1}(2)$ (B) $\operatorname{cosec}^{-1}\left(\sqrt{2}\right)$ (C) $\cot^{-1}(0)$ (D) $\tan^{-1}\left(\sqrt{2}+1\right)$

3. The value of $\cot\frac{7\pi}{16} + 2\cot\frac{3\pi}{8} + \cot\frac{15\pi}{16}$ is

(A) – 2 (B) – 3 (C) – 4 (D) – 6

4. The variable 'x' stisfying the equation

$|\sin x \cos x| + \sqrt{2+\tan^2 x+\cot^2 x} = \sqrt{3}$, belongs to the interval

(A) $\left[0, \frac{\pi}{3}\right]$ (B) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (C) $\left[\frac{3\pi}{4}, \pi\right)$ (D) non existent

5. If $x = \cos\alpha + \cos\beta - \cos(\alpha+\beta)$ and $y = 4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\left(\frac{\alpha+\beta}{2}\right)$, then (x – y) equals

(A) 0 (B) 1 (C) – 1 (D) – 2

6. The expression $(\alpha\tan\gamma + \beta\cot\gamma)(\alpha\cot\gamma + \beta\tan\gamma) - 4\alpha\beta\cot^2 2\gamma$ is

(A) independent of α, β (B) independent of γ

(C) dependent on γ (D) dependent on α, β

7. The value of $x \in (0, 90°)$ and satisfying $\cos x° = \sin 61° + \sin 47° - \sin 25° - \sin 11°$, is

(A) 7° (B) 11° (C) 13° (D) 17°

8. Let $f(\theta) = \sqrt{\sin^4\theta + 4\cos^2\theta} - \sqrt{\cos^4\theta + 4\sin^2\theta}$, then the value of $f\left(11\frac{1°}{4}\right)$ is equal to

(A) $\frac{\sqrt{2-\sqrt{2}}}{2}$ (B) $-\frac{\sqrt{2+\sqrt{2}}}{2}$ (C) $-\frac{\sqrt{2-\sqrt{2}}}{2}$ (D) $\frac{\sqrt{2+\sqrt{2}}}{2}$

9. If $\frac{\cos^6 9° + \sin^6 9°}{(\cos^2\theta + \sin^2\theta)^3 - 3\sin^2\theta\cos^2\theta} \equiv \frac{a + b\sqrt{c}}{32}$ where a, b, c ∈ N then (a + b + c) equals

(A) 25 (B) 28 (C) 31 (D) none

10 In triangle ABC, if $2\Delta^2 = \frac{a^2b^2c^2}{a^2+b^2+c^2}$ then the triangle ABC, is

(All symbols used have their usual meaning in a triangle.)

(A) right angled.
(B) isosceles but not right angled.
(C) isosceles right angled.
(D) equilateral.

More then one are correct

11. The expression $\sqrt{\sin^4(37.5)° + 4\cos^2(37.5)°} + \sqrt{\cos^4(37.5)° + 4\sin^2(37.5)°}$ simplifies to

(A) an irrational number
(B) a prime number
(C) a natural number which is not composite
(D) a real number of the form $a + \sqrt{b}$ where a and b are prime.

12. Let $f_n(\theta) = \sum_{n=0}^{n} \frac{1}{4^n}\sin^4(2^n\ \theta)$. Then which of the following alternative(s) is/are correct ?

(A) $f_2\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$
(B) $f_3\left(\frac{\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$
(C) $f_4\left(\frac{3\pi}{2}\right) = 1$
(D) $f_5(\pi) = 0$

13. If $f(\theta) = \sum_{n=1}^{6} \text{cosec}\left(\theta + \frac{(n-1)\pi}{4}\right)\text{cosec}\left(\theta + \frac{n\pi}{4}\right)$, where $0 < \theta < \frac{\pi}{2}$, then minimum value of f

(A) lies between 3 and 4
(B) lies between 2 and 3
(C) occures when $\theta = \frac{\pi}{4}$
(D) occures when $\theta = \frac{\pi}{6}$

14 Let $E = \cos^2\frac{\pi}{7} + \cos^2\frac{2\pi}{7} + \cos^2\frac{3\pi}{7}$. Then which of the following alternative(s) is/are incorrect?

(A) $\frac{1}{2} < E < \frac{3}{4}$ (B) $\frac{3}{4} < E < 1$ (C) $1 < E < \frac{3}{2}$ (D) $\frac{3}{2} < E < \frac{7}{4}$

15. Which of the following expression(s) have their value equal to unity?

(A) $\cos^4\theta + 2\sin^2\theta - \sin^4\theta$
(B) $\sin^2\theta\cos^2\phi + \cos^2\theta\sin^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta\cos^2\phi$
(C) $\sin(285° - \theta)\cos(\theta + 165°) + \sin(\theta + 165°)\cos(\theta - 285°)$
(D) $\frac{\sin^2\theta}{2}(1 + \cot^2\theta) + \frac{\cos^2\theta}{2}(1 + \tan^2\theta)$

16. Which of the following identities wherever defined hold(s) good?

(A) $(\tan\theta + \cot\theta)^2 = \sec^2\theta \ \text{cosec}^2\theta$

(B) $\tan 3\theta - \tan 2\theta - \tan\theta = \tan 3\theta \tan 2\theta \ \tan\theta$

(C) $\dfrac{\cot^2\theta + 1}{1 - \sin^2\theta} = \sec^2\theta + \text{cosec}^2\theta$

(D) $\tan\left(\dfrac{\pi}{4}+\theta\right) + \tan\left(\dfrac{\pi}{4}-\theta\right) = 2\ \text{cosec}2\theta$

17. The equation $x^3 - \dfrac{3}{4}x = -\dfrac{\sqrt{3}}{8}$ is satisfied by

(A) $x = \cos\left(\dfrac{5\pi}{18}\right)$ (B) $x = \cos\left(\dfrac{7\pi}{18}\right)$ (C) $x = \cos\left(\dfrac{23\pi}{18}\right)$ (D) $x = \cos\left(\dfrac{17\pi}{18}\right)$

18. The value of $\sin 27° - \cos 27°$ is equal to

(A) $-\dfrac{\sqrt{3-\sqrt{5}}}{2}$ (B) $-\dfrac{\sqrt{5-\sqrt{5}}}{2}$ (C) $-\dfrac{\sqrt{5}-1}{2\sqrt{2}}$ (D) $\dfrac{\sqrt{3-\sqrt{5}}}{2}$

19. In a ΔABC, if $r = 1, R = 3, s = 5$, then which of the following is/are correct?

(A) Area of Δ ABC is 5.

(B) Product of the sides of the Δ ABC is 60.

(C) $a^2 + b^2 + c^2 = 24$

(D) Sum of the ex-radii of Δ ABC is 13

20. Which of the following pair(s) of function have same graphs?

(A) $f(x) = \dfrac{\sec x}{\cos x} - \dfrac{\tan x}{\cot x}$, $g(x) = \dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\text{cosec}\, x}$

(B) $f(x) = \text{sgn}(x^2 - 4x + 5)$, $g(x) = \text{sgn}\left(\cos^2 x + \sin^2\left(x + \dfrac{\pi}{3}\right)\right)$ where sgn denotes signum function.

(C) $f(x) = e^{\ln(x^2+3x+3)}$, $g(x) = x^2 + 3x + 3$

(D) $f(x) = \dfrac{\sin x}{\sec x} + \dfrac{\cos x}{\text{cosec}\, x}$, $g(x) = \dfrac{2\cos^2 x}{\cot x}$

21. If $2\tan^2\theta_1 \tan^2\theta_2 \tan^2\theta_3 + \tan^2\theta_1 \tan^2\theta_2 + \tan^2\theta_2 \tan^2\theta_3 + \tan^2\theta_3 \tan^2\theta_1 = 1$, then which of the following relations holds good ?

(A) $\sin^2\theta_1 + \sin^2\theta_2 + \sin^2\theta_3 = 1$

(B) $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3 = 1$

(C) $\sin^2\theta_1 + \sin^2\theta_2 + \sin^2\theta_3 = 2$

(D) $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3 = -1$

22. The base BC of ΔABC is fixed and the vertex A moves, satisfying the condition $\cot\dfrac{B}{2} + \cot\dfrac{C}{2} = 2\cot\dfrac{A}{2}$, then

(A) $b + c = a$

(B) $b + c = 2a$

(C) vertex A moves on a straight line.

(D) vertex A moves on an ellipse.

23. In ΔABC, D is a point on BC such that DB = 14, DA = 13 and DC = 4. If the circumcircle of the Δ ADB is congruent to the circumcircle of the Δ ADC then which of the following is/are correct?

(A) angle B > 45° but angle C < 45°

(B) both the angles B and C are greater than 45°

(C) area of the triangle is 108 sq. units

(D) measure of angle A equal to $\tan^{-1}\left(\dfrac{24}{7}\right)$

24. In ΔABC, if $\cos A + \cos B = 4\sin^2\frac{C}{2}$, then which of the following hold(s) good?

(A) $\cot\frac{A}{2}\cot\frac{B}{2} = 2$
(B) $\cot\frac{A}{2}\cot\frac{B}{2} = 3$
(C) a, c, b are in A.P.
(D) a, b, c are in G.P.

25. In a Δ AEX, T is the mid point of XE, and P is the mid point of ET. If the ΔAPE is equilateral of side length equal to unity then which of the following alternative(s) is/are correct?

(A) $AX = \sqrt{13}$
(B) $\angle EAT = 90°$
(C) $\cos\angle XAE = \frac{-1}{\sqrt{13}}$
(D) $AT = \frac{1}{\sqrt{3}}$

26. Which of the following expression(s) have their value equal to four times the area of the triangle ABC? (All symbols used have their usual meaning in a triangle)

(A) $rs + r_1(s-a) + r_2(s-b) + r_3(s-c)$
(B) $\dfrac{(a+b+c)^2}{\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}}$
(C) $(a^2 + b^2 - c^2)\tan B$
(D) $b^2\sin 2C + c^2\sin 2B$

27. The expression $\cos^2(\alpha+\beta+\gamma) + \cos^2(\beta+\gamma) + \cos^2\alpha - 2\cos\alpha\cos(\beta+\gamma)\cos(\alpha+\beta+\gamma)$, is

(A) independent of α
(B) independent on β
(C) dependent on γ only
(D) dependent on α, β and γ

28. If $\frac{\sin^4 x}{5} + \frac{\cos^4 x}{4} = \frac{1}{9}$, then which of the following **is/are TRUE?**

(A) $\cot^2 x = \frac{4}{5}$
(B) $\tan^2 x = \frac{4}{5}$
(C) $\frac{64}{\cos^6 x} + \frac{125}{\sin^6 x} = 1458$
(D) $\frac{125}{\cos^6 x} + \frac{64}{\sin^6 x} = 1458$

29. Let $f_n(\theta) = \sum_{n=1}^{n}\tan\left(\frac{\theta}{2^n}\right)\sec\left(\frac{\theta}{2^{n-1}}\right)$

Then which of the following alternative(s) is/are correct ?

(A) $f_3(2\pi) = -1$
(B) $f_4\left(\frac{4\pi}{3}\right) = 2(\sqrt{3}-1)$
(C) $f_5(4\pi) = \sqrt{2}-1$
(D) $f_6(48\pi) = 1$

30. Which of the following trigonometric ratio's can be equal to $\left(\log_{0.\bar{5}}(1.8) + \log_{1.8}(0.\bar{5})\right)$ for some value of θ

(A) $\cot\theta$
(B) $\cos\theta$
(C) $\sec\theta$
(D) $\text{cosec}\,\theta$

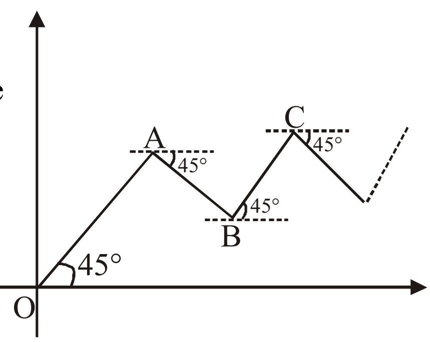
Subjective

31. Let a, b, c, d be real numbers such that a + b + c + d = 10, then the minimum value of $a^2 \cot 9° + b^2 \cot 27° + c^2 \cot 63° + d^2 \cot 81°$ is $\sqrt{n}$ $(n \in N)$. Find n.

32. Let $a = \sum_{r=1}^{11} \tan^2\left(\frac{r\pi}{24}\right)$ and $b = \sum_{r=1}^{11} (-1)^{r-1} \tan^2\left(\frac{r\pi}{24}\right)$ then find the value of $\log_{(2b-a)}(2a-b)$.

33. Let A denotes the value of expression $4\left(\cos\frac{2\pi}{15} + \cos\frac{4\pi}{15} - \cos\frac{7\pi}{15} - \cos\frac{\pi}{15}\right)$ and B denotes the value of $8 \cot(\alpha + \beta + \gamma)$, where $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are the real roots of the cubic $x^3 - 8(a-b)x^2 + (2a-3b)x - 4(b+1) = 0$. Find absolute value of (AB).

34. Points O, A, B, C ...... are shown in figure where OA = 2AB = 4BC = ... so on. If A is the centroid of a triangle whose orthocentre and circumcenter are (2, 4) and $\left(\frac{7}{2}, \frac{5}{2}\right)$ respectively. If an insect starts moving from the point O(0, 0) along the straight line in zig-zag fashion and terminates ultimately at point $P(\alpha, \beta)$ then find the value of $(\alpha + \beta)$



35. If $7\alpha = 2\pi$, then find the absolute value of the expression $y = \sec\alpha + \sec 2\alpha + \sec 4\alpha$.

36. If $\sin\alpha = \frac{12}{37}$, $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\cos\beta = \frac{20}{101}$, $\beta \in \left(\frac{3\pi}{2}, 2\pi\right)$ then the value of $\text{cosec}(\alpha + \beta)$ can be expressed in the lowest form as $\frac{p}{q}$ $(p, q \in N)$ then find the value of $(p + q)$.

37. Compute the square of the value of the expression $\frac{4 + \sec 20°}{\text{cosec } 20°}$.

38. Let $\cot\alpha = 2p + 3$ and $\cot\beta = 2p + 1$ where p is a constant. If $\text{cosec}^2\alpha - \text{cosec}^2\beta = 24$, then the value of $\tan(\alpha + \beta)$ can be expressed in the lowest form as $\frac{m}{n}$ where $m, n \in N$. The value of $(m + n)$ equals

39. If $S_n = \left(1 - \tan^4\frac{\pi}{2^3}\right)\left(1 - \tan^4\frac{\pi}{2^4}\right)........\left(1 - \tan^4\frac{\pi}{2^n}\right)$ then find $\lim_{n\to\infty} S_n$.

40. The triangle ABC, right angled at C, has median AD, BE and CF. AD lies along the line $y = x + 3$, BE lies along the line $y = 2x + 4$. If the length of the hypotenuse is 60, find the area of the triangle ABC.

ANSWER KEY

1.	(A)	2.	(C)	3.	(C)	4.	(D)	5.	(B)	6.	(BD)
7.	(A)	8.	(D)	9.	(C)	10.	(A)	11.	(BC)	12.	(CD)
13.	(BC)	14.	(ABD)	15.	(ABCD)	16.	(ABC)	17.	(ABD)	18.	(AC)
19.	(ABCD)	20.	(ABCD)	21.	(AB)	22.	(BD)	23.	(BCD)	24.	(BC)
25.	(ABC)	26.	(ABD)	27.	(AB)	28.	(AC)	29.	(ABD)	30.	(ACD)
31.	[125]	32.	[2]	33.	[4]	34.	[8]	35.	[4]	36.	[7442]
37.	[3]	38.		39.		40.	[400 sq. units]				