MATHS PAPER - I (SOLUTION)

41. Ans. (b)

If a, b,c are in A.P.

$2b = a + c$,$\therefore$ $a + c$ = even

a & c are both odd and even

$\therefore$ No. of ways of selection = ${}^{12}C_2 + {}^{12}C_2 = 132$

42. Ans. (a)

Let roots be a – d, a, a + d, then

$(a - d) + a + (a + d) = -\frac{(-p)}{1} \Rightarrow 3a = p$

a = p/3 will satisfy the equation

$\left(\frac{p}{3}\right)^3 - P\left(\frac{p}{3}\right)^2 + 9\frac{p}{3} - r = 0 \Rightarrow 2p^3 - 9pq + 27r = 0$

43. Ans. (b)

$f(x) + f(y) = f(x + y + 1) - 2\sqrt{f(x)f(y)}$, $f(0) = 1$

$f(x) + f(y) + 2\sqrt{f(x)f(y)} = f(x + y + 1)$

$\left(\sqrt{f(x)} + \sqrt{f(y)}\right)^2 = f(x + y + 1) \Rightarrow \sqrt{f(x)} + \sqrt{f(y)} = \sqrt{f(x + y + 1)}$

$y = 0$, $\sqrt{f(x)} + \sqrt{f(0)} = \sqrt{f(x+1)} \Rightarrow \sqrt{f(x+1)} - \sqrt{f(x)} = 1$

Put $x = 0$, $\sqrt{f(1)} - \sqrt{f(0)} = 1 \Rightarrow f(1) = 4$

$x = 1$; $\sqrt{f(2)} - \sqrt{f(1)} = 1 \Rightarrow f(2) = 9$

Similarly $f(x) = (x + 1)^2$, $f'(x) = 2(x + 1)$

$f'(100) = 202$

44. Ans. (a)

$\frac{\tan A}{1} = \frac{\tan B}{2} = \frac{\tan C}{3} = k$

Also, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$k + 2k + 3k = k \cdot 2k \cdot 3k \Rightarrow k = 1$ ($k \neq 0$)

$\therefore$ $\tan A = 1$, $\tan B = 2$, $\tan C = 3$

$\sin A = \frac{1}{\sqrt{2}}, \sin B = \frac{2}{\sqrt{5}}, \sin C = \frac{3}{\sqrt{10}}$

$$\frac{\sin A}{\frac{1}{\sqrt{2}}} = \frac{\sin B}{\frac{2}{\sqrt{5}}} = \frac{\sin C}{\frac{3}{\sqrt{10}}} \qquad \Rightarrow 6\sqrt{2}\,a = 3\sqrt{5}\,b = 2\sqrt{10}\,C$$

45. Ans.(d)

for unique solution $\Delta \neq 0$

$$\begin{vmatrix} a & b & a \\ b & b & b \\ c & c & a \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} a-b & b & a \\ 0 & b & b \\ 0 & c & a \end{vmatrix} \neq 0 \qquad (c_1 \to c_1 - c_2)$$

$(a – b)(ab – bc) \neq 0$, $b(a – b)(a – c) \neq 0$

$b \neq 0$, $a \neq b$, $a \neq c$

If b = 1, (a, c) = (2, 0) (0,2) (2,1) (0, 2)

If b = 2, (a,c) = (0,1) (0,2) (1,0) (1,2)

favourable No. of ways = 8

total no. of ways = 3 × 3 × 3 = 27

probability = $\frac{8}{27}$

46. Ans. (a)

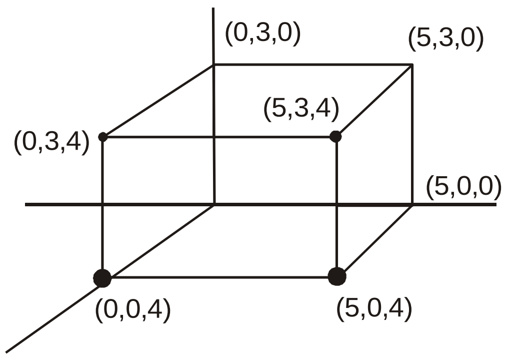
vector parallel to normal to the plane AGD

$\vec{n}_1 = (5\hat{i} + 4\hat{k}) \times 3\hat{j} = 15\hat{k} - 12\hat{i}$

vector parallel to normal to the plane BCF is

$\vec{n}_2 = 3\hat{i} \times (5\hat{i} + 3\hat{j} - 4\hat{k}) = -15\hat{k} - 12\hat{i}$

$\cos\theta = \frac{\vec{n}_1 . \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{-9}{41}$, Angle = $\cos^{-1}\frac{9}{41}$



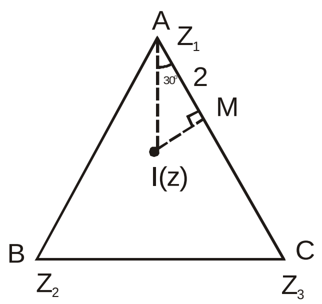
47. Ans. (a)

$|z – z_1| = AI$

$\cos 30^0 = \frac{AM}{AI} \Rightarrow \frac{\sqrt{3}}{2} = \frac{2}{AI}$

$\Rightarrow AI = \frac{4}{\sqrt{3}}$

$|z – z_1| = 4/\sqrt{3}$



48. Ans. (d)

$$\frac{x^2}{16} + \frac{y^2}{4} = 1 \qquad \Rightarrow y = mx \pm\sqrt{16m^2 + 4} \qquad \ldots(1)$$

$y^2 = 4(x + 1) \quad \Rightarrow \quad y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m} \qquad \ldots (2)$

for common slope $\sqrt{16m^2 + 4} = m + \frac{1}{m}$

$16m^2 + 4 = m^2 + \frac{1}{m^2} + 2 \Rightarrow 15m^2 + 2 = \frac{1}{m^2}$

$15m^4 + 2m^2 - 1 = 0$, $\therefore$ equation is $15x^4 + 2x^2 - 1 = 0$

49. Ans. (a)

Let equation of the plane be

$x + y + lz = 1$ ($\because$ x intercept =1 = y intercept)

Angle with x + y = 3 is $\pi/4$

$$\therefore \cos\frac{\pi}{4} = \left(\frac{1+1+0}{\sqrt{1^2+1^2+l^2}\sqrt{1^2+1^2}}\right) \Rightarrow \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{l^2+2}.\sqrt{2}}$$

$l = \pm\sqrt{2}$ $\therefore$ direction ratios are 1, 1, $\sqrt{2}$

50. Ans. (d)

$z = \frac{\omega + 5i}{4 - 2\omega}$ is purely real , let $\omega = x + iy$

$\frac{x + iy + 5i}{4 - 2x - 2iy}$ = purely real, $\therefore$ Imaginary part = 0

$$\text{Im}\left[\frac{x + i(y+5)}{4 - 2x - 2iy} \times \frac{4 - 2x + 2iy}{4 - 2x + 2iy}\right] = 0$$

$(y + 5)(4 - 2x) + 2xy = 0 \quad \Rightarrow \quad 4y - 2xy + 20 - 10x + 2xy = 0$

$-10x + 4y + 20 = 0 \quad \Rightarrow \quad 5x - 2y - 10 = 0$

51. Ans. (a) & (b)

divide by x^2, $x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$

$$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0$$

let $x + \frac{1}{x} = t$; $x^2 + \frac{1}{x^2} = t^2 - 2$

$t^2 - 2 - 10t + 26 = 0, \qquad \Rightarrow t^2 - 10t + 24 = 0$

$t = 6, 4$

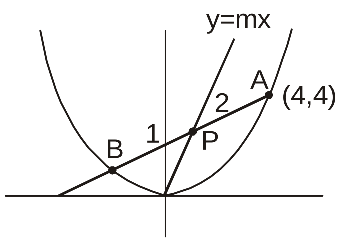
$x+\frac{1}{x}=6$ or $x+\frac{1}{x}=4$

$x^2-6x+1=0$, $x^2-4x+1=0$

$x=3\pm 2\sqrt{2}$, $x=2\pm\sqrt{3}$

All roots are real and irrational

52. Ans. (b) & (c)



Let p (α, mα) Let B (x, y)

$$\left(\frac{2x+4}{3},\frac{2y+4}{3}\right)=(\alpha,m\alpha)$$

$$(x, y)\equiv\left(\frac{3\alpha-4}{2},\frac{3m\alpha-4}{2}\right)$$

$\because$ pt is on $x^2=4y$

$$\left(\frac{3\alpha-4}{2}\right)^2=4\left(\frac{3m\alpha-4}{2}\right)$$

$9\alpha^2+16-24\alpha=24m\alpha-32$

$9\alpha^2-24\alpha(1+m)+48=0$

$3\alpha^2-8\alpha(1+m)+16=0$

for 2 chords D > 0, $64(1+m)^2>4.3.16$

$m>\sqrt{3}-1$ or $m<-\sqrt{3}-1$

53. Ans. (a)& (c)

Here A' = A , B' = – B

$(A+B)^2=A^2+B^2+2AB$

$A^2+B^2+AB+BA=A^2+B^2+2AB\Rightarrow AB=BA$

(AB)' + AB = B' A' + AB = (– BA) + AB

= – AB + AB = 0

|(AB)' + AB | = 0 $\Rightarrow$ |(AB)' – AB| $\neq$ 0

(AB)' = – AB

54. Ans. (b)

$x\vec{a}+y\vec{b}=2\vec{a}+3\vec{b}$

$\vec{b}(y-3)=\vec{a}(2-x)$

$$\left|\frac{\vec{a}}{\vec{b}}\right|=\left|\frac{y-3}{2-x}\right|$$

$\because$ $\vec{a},\vec{b}$ are collinear, $\vec{a}=\lambda\vec{b}\Rightarrow$ so, we can't compare coefficient of $\vec{a},\ \vec{b}$

55. (a) & (b)

$\sin x = f^{1}(x) + f^{II}(x) + f^{III}(x) + ...\infty$

diff. w.r.t. x, $\cos x = f^{II}(x) + f^{III}(x) + f^{iv}(x) + ...$

$\cos x = \sin x - f^{1}(x)$

$f^{1}(x) = \sin x - \cos x,$ $\quad \therefore f^{II}(x) = \cos x + \sin x$

$f^{III}(x) = -\sin x + \cos x$ $\quad \therefore f^{iv}(x) = -\cos x - \sin x$

$f^{1}(x) + f^{II}(x) + f^{III}(x) + f^{iv}(x) = 0$

$$\therefore \sum_{r=1}^{100} f^{r}(n) = 0 \ ; \ \sum_{r=1}^{101} f^{r}(x) = \sum_{r=1}^{100} f^{r}(x) + f^{101}(x)$$

$$\sum_{r=1}^{101} f^{r}(x) = f'(x) = \sin x - \cos x = \sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$$

56. 9

Let equation of plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$(1,1,1) \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

GM $\geq$ HM, volume of tetrahedron = $\frac{1}{6}\alpha\beta\gamma$, $\sqrt[3]{\alpha\beta\gamma} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$

$\sqrt[3]{\alpha\beta\gamma} \geq 3. \Rightarrow \alpha\beta\gamma \geq 27$

$v = \frac{1}{6}\alpha\beta\gamma \geq \frac{27}{6}$

$\Rightarrow v \geq 9/2$

$2v \geq 9$

57. 6

$3x - 4y + k = 0$ lies outside $x^2 + y^2 - 2x - 2y + 1 = 0$

c(1,1), r = 1

$\left|\frac{3 - 4 + k}{5}\right| > 1 \quad \Rightarrow \quad k > 6$ or $k < -4$

$\because$ $3x - 4y + k = 0$ cuts +ve x-axis, $x_{int} > 0, k < 0$

$k < -4$ …(i)

Similarly $3x - 4y + k = 0$ lies outside $x^2 + y^2 - 16x - 2y + 61 = 0$

C (8, 1) ; r = 2

$\left|\frac{24 - 4 + k}{5}\right| > 2 \quad \Rightarrow \quad |k + 20| > 10$

$k > -10$ or $k < -30$ …(2)

$x_{int} < 8 \quad \frac{-k}{3} < 8 \quad \Rightarrow k > -24$

by (1) and (2) k $\in$ (–10, – 4), a = – 10, b = 4

58. Ans. 2

$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$; $a_r = r^2 \times \left(\frac{100-r+1}{r}\right)$

$a_r = r\,(101 - r)$

coefficient x^{99} = – ($a_1 + a_2 + ... + a_{100}$)

$$\lambda = -\sum_{r=1}^{100}(101r - r^2) = -\left[101.\frac{100.101}{2} - \frac{100.101.201}{6}\right]$$

$$= -100.101\left[\frac{101}{2} - \frac{201}{6}\right] = -171700$$

$\frac{-\lambda}{17} = 10100$, sum of digit = 2

59. 5

f(x) has local minimum at x =1

$f(1) \le \lim_{x\to 1^+} (2x+3)$

$f(1) \le 5$, $\Rightarrow$ $a \le 5$ Also $a \ge 5$, $\therefore$ a = 5

60. 4

$f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\log t}{1+t}dt + \int_1^{1/e}\frac{\log t}{1+t}dt$ put $t = \frac{1}{u}$ in II integral

$I_2 = \int_1^{1/e}\frac{\log t}{1+t}dt$, $I_2 = \int_1^e \frac{-\log u}{1+\frac{1}{u}} \times \frac{-1}{u^2}du = \int_1^e \frac{\log u}{u(u+1)}\,du = \int_1^e \frac{\log t}{t(t+1)}dt$

$f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\log t}{1+t}dt + \int_1^e \frac{\log t}{(1+t)t}\,dt$

$$= \int_1^e \frac{\log t}{1+t}\left[1+\frac{1}{t}\right]dt = \int_1^e \frac{\log t}{t}dt = \left[\frac{(\log t)^2}{2}\right]_1^e$$

$\therefore$ $f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2}$ $\Rightarrow$ $8\left(f(e) + f\left(\frac{1}{e}\right)\right) = 4$