PAPER - II (SOLUTION)

41. Ans. (c)

$$\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}\,(bx-\sin x)}=1$$

$$\Rightarrow \quad \lim_{x\to 0}\frac{x^3}{\sqrt{a}\,(bx-\sin x)}=1 \qquad \Rightarrow \quad \lim_{x\to 0}\frac{3x^2}{\sqrt{a}\,(b-\cos x)}=1$$

$\therefore$ b – 1 = 0 i.e. b = 1

$$\therefore \quad 1=\lim_{x\to 0}\frac{3x^2}{\sqrt{a}\,(1-\cos x)} = \lim_{x\to 0}\frac{3x^2}{\sqrt{a}\,2\sin^2\frac{x}{2}} = \lim_{x\to 0}\frac{6}{\sqrt{a}}\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2 = \frac{6}{\sqrt{a}}$$

$\Rightarrow$ a = 36

thus a + 2b = 38

42. Ans. (c)

$g'(x^2) = x^3$

$\Rightarrow$ $g'(t) = t^{\frac{3}{2}}$, where $x^2 = t$

$\Rightarrow$ $g(t) = \frac{t^{5/2}}{5/2}+C$ $\therefore$ $g(1) = \frac{(1)^{5/2}}{5/2}+C$ $\therefore$ $C = \frac{3}{5}$

$\therefore$ $g(t) = \frac{(t)^{5/2}}{5/2}+\frac{3}{5}$, $g(4) = \frac{(4)^{5/2}}{5/2}+\frac{3}{4}=\frac{67}{5}$

43. Ans. (c)

$$I=\int\frac{\cos^4 x\,dx}{\sin^3 x(\sin^5 x+\cos^5 x)^{3/5}} \quad =\int\frac{\cos^4 x\,dx}{\sin^6 x(1+\cot^5 x)^{3/5}}$$

$= \int\frac{\cot^4 x\,\mathrm{cosec}^2 x\,dx}{(1+\cot^5 x)^{3/5}}$ put $1 + \cot^5 x = t$ $\quad 5\cot^4 x\,\mathrm{cosec}^2 x\,dx = -dt$

$$= -\frac{1}{5}\int\frac{dt}{t^{3/5}} = -\frac{1}{5}\left(\frac{t^{-\frac{3}{5}+1}}{-\frac{3}{5}+1}\right) + C = -\frac{1}{2}(1+\cot^5 x)^{2/5} + C$$

$\therefore$ A = 5, B = $\frac{2}{5}$ and AB = 2

44. Ans. (b)

$$p = \lim_{n\to\infty}\left[\frac{\prod_{r=1}^{n}(n^3+r^3)}{n^{3n}}\right]^{1/n} \qquad \ell n p = \lim_{n\to\infty}.\frac{1}{n}\sum_{r=1}^{n}\ell n\left(1+\left(\frac{r}{n}\right)^3\right) = \int_0^1 \ell n(1+x^3)\,dx = \ell n\,2 - 3 + 3\lambda\ .$$

45. Ans. (b)

f(x) = sin x

f(x) + f(π – x) = 2

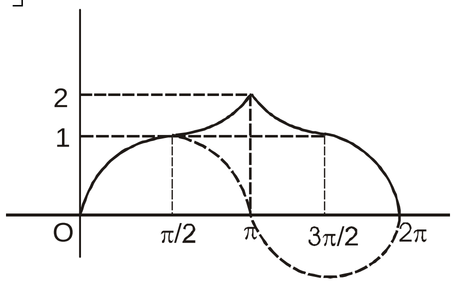
$f(x) = 2 - f(\pi - x) = 2 - \sin(\pi - x) = 2 - \sin x \qquad x \in \left(\frac{\pi}{2}, \pi\right]$

f(x) = f (2π – x)

∴ f(x + π) = f (π – x)

So curve is symmetric w.r.t. line x = π for (π , 2π]

f(x) = f(2π – x) = – sinx



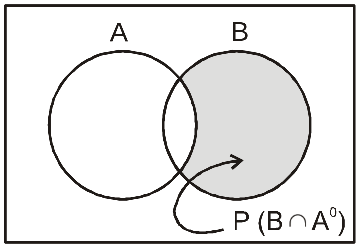
$$\text{Area} = 2\left(\int_0^{\pi/2} \sin x\, dx \int_{\pi/2}^{\pi} (2 - \sin x)\, dx\right) = 2\left(1 + 2 \times \frac{\pi}{2} - 1\right) = 2\pi$$

46. Ans. (b)

$P(A) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$

$$P\left(\frac{B}{A^C}\right) = \frac{P(B \cap A^C)}{P(A^C)}$$

$$= \frac{P(A \cup B) - P(A)}{1 - P(A)} = \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$$



47. Ans. (a)

$12 \sin x + 5 \cos x \le 13$ and $2y^2 - 8y + 21 \ge 13$

Equality holds if 12 sinx + 5 cos x = 13 and y = 2

$\cot x = \frac{5}{12}$ and y = 2

Hence $12 \cot\left(\frac{xy}{2}\right)$

$= 12. \cot\left(\frac{2x}{2}\right) = 12 \cot x = 12 . \frac{5}{12} = 5$

48. Ans. (b)

$0 \le x^2 + x + 1 \le 1$ and $0 \le x^2 + x \le 1$

∴ x = –1, 0

for x = –1

$\text{L.H.S} = 2 \sin^{-1} 1 + \cos^{-1} 0 = \frac{3\pi}{2}$

∴ x = – 1 is a solution, for x = 0 , L.H. S = $2 \sin^{-1} 1 + \cos^{-1} 0 = \frac{3\pi}{2}$

∴ x = 0 is a solution

∴ sum of the solution = – 1

49. Ans. (b)

∵ PQ = PR i.e. parallelogram PQRS is a rhombus

∴ Mid point of QR = Midpoint of PS and QR ⊥ PS

∴ S is the mirror image of P w.r.t QR

$\because$ $L \equiv 2x + y = 6$ Let $P \equiv (k, 6 - 2k)$

$\because$ $\angle PQO = \angle PRO = \frac{\pi}{2}$

$\therefore$ OP is diameter of circumcircle PQR, then centre is $\left(\frac{k}{2}, 3-k\right)$

$\therefore$ $x = \frac{k}{2}$ $\Rightarrow$ $k = 2x$

$y = 3 - k$ $\therefore$ $2x + y = 3$

50. Ans. (d)

P(6, 8)

$\therefore$ equation of QR is $6x + 8y = 4$ $\Rightarrow$ $3x + 4y - 2 = 0$

$\therefore$ $PM = \frac{48}{5}$ and $PQ = \sqrt{96}$

$QM = \sqrt{96 - \frac{(48)^2}{25}} = \sqrt{\frac{96}{25}}$ $\therefore$ $QR = 2\sqrt{\frac{96}{25}}$

$\therefore$ Area of $\Delta PQR = \frac{1}{2}$. PM . QR $= \frac{196\sqrt{6}}{25}$

$\because$ PQRS is a rhombus

$\therefore$ Area of ΔQRS = Area of $\Delta PQR = \frac{196\sqrt{6}}{25}$ sq. units

51. Ans. (a)

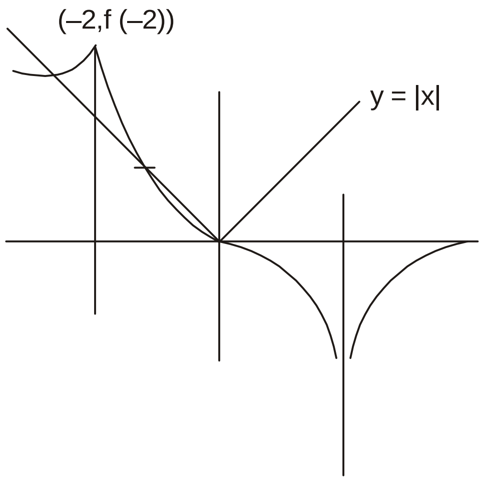
Product of the length of the perpendicular segments from the focii on tanent at P(4, 7) is $= b^2 = 20$

52. Ans. (b)

Locus of mid point of QR is another ellipse having the same eccentricity as that of ellipse (e).

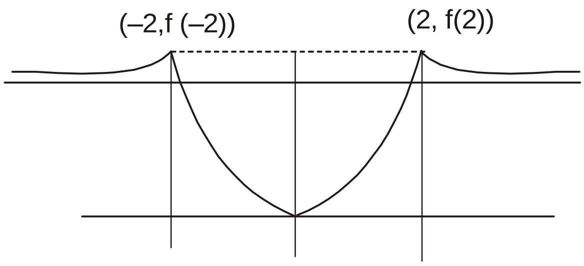
$\Rightarrow$ $e' = e = \frac{\sqrt{5}}{3}$

53. Ans. (c)



Three points of intersection. Three solutions

54. (b)



55. (a)

The diagonals are $\vec{d}_1 = 3\hat{a} - 2\hat{b} + 2\hat{c} + (-\hat{a} - 2\hat{c}) = 2\hat{a} - 2\hat{b}$

$\vec{d}_2 = 3\hat{a} - 2\hat{b} + 2\hat{c} - (-\hat{a} - 2\vec{c}) = 4\hat{a} - 2\vec{b} + 4\hat{c}$

Angle between them $= \cos^{-1}\dfrac{\vec{d}_1.\vec{d}_2}{|\vec{d}_1|.|\vec{d}_2|} = \cos^{-1}\left(\dfrac{8+4}{2\sqrt{2}(6)}\right) = \cos^{-1}\dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$

56. Ans. (d)

$\vec{x} + \vec{y} = 2\hat{b} - 3\hat{c}$ and $\vec{y} + \vec{z} = -2\hat{a} + 3\hat{b} - 3\hat{c}$

$$\therefore \quad (\vec{x} + \vec{y}) \times (\vec{y} + \vec{z}) = \begin{vmatrix} \hat{a} & \hat{b} & \hat{c} \\ 0 & 2 & -3 \\ -2 & 3 & -3 \end{vmatrix} = 3\hat{a} + 6\hat{b} + 4\hat{c}$$

57. Ans. (A) → (s), (B) → (p) (C) → (q), (D) → (s)

(A) equation of the circle is $(x - r - 1)^2 + y^2 = r^2$

putting $y^2 = 4x$ and then D = 0

(B) AB is focal chord.

Min AB = latus rectum = 4a = 16

(C) Shortest distance along common normal slope of common normal = 0, ± 2

So feet of normals will be (4, 4) & (5, 2) or (4,– 4) and (5, – 2)

$\therefore \quad d = \sqrt{1+4} = \sqrt{5}$

(D) Harmonic mean = 2a = 4

58. Ans. (A) $\rightarrow$ (q), (B) $\rightarrow$ (p) (C) $\rightarrow$ (s), (D) $\rightarrow$ (p)

(A) $\displaystyle\int_{6\pi-3}^{6\pi} \frac{\cos(x/3)}{6\pi + 3 - x}\,dx$

Let $6\pi - 3t = x$

$$\int_0^1 \frac{3\cos(2\pi - t)}{3(1+t)}dt = \int_0^1 \frac{\cos t}{1+t}\,dt = \int_0^1 \frac{\cos x}{1+x}dx = k$$

(B) $\displaystyle\int_{-1}^{1}\left(\sin^{-1}\left[x+\frac{3}{4}\right]\right)dx = \int_{1}^{3/4}\left(-\frac{\pi}{2}\right)dx + \int_{-3/4}^{1/4} 0\,dx + \int_{1/4}^{1}\frac{\pi}{2}\,dx$

$$= -\frac{\pi}{8} + \frac{\pi}{2}\times\frac{3}{4} = \frac{2\pi}{8} = \frac{\pi}{4}$$

(C) $\because \quad -|x| \le [x]$ for all + ve x

$\quad\quad -|x| \ge [x]$ for all – ve x

$$\Rightarrow \int_{-3}^{3} f(x)\,dx = \int_{-3}^{0}(x - |x|)\,dx + \int_{0}^{3}(x + [x])\,dx$$

$$= \int_{-3}^{0} 2x\,dx + \int_{0}^{3} x\,dx + \int_{0}^{3}[x]\,dx = -\frac{3}{2}$$

59. Ans. (A) $\rightarrow$ (s), (B) $\rightarrow$ (r) (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

(A) $5^{2+4+6+.......+2x} = (25)^{28}$

$\Rightarrow 5^{x(x+1)} = 5^{56}$

$\Rightarrow x^2 + x - 56 = 0 \Rightarrow x = 7$ as $x > 0$

(B) $2\log_5 x = \log_{\sqrt{5}}\left(\dfrac{1/4}{1-1/2}\right)\log_5(0.2)$

$$= \log_{\sqrt{5}}(1/2)\log_5\left(\frac{1}{5}\right)$$

$$= -\frac{\log_5(1/2)}{\log_5\sqrt{5}} = \log_5 4$$

$\Rightarrow x = 2$

(C) $\log x = \log_{2.5}\left(\dfrac{1/3}{1-1/3}\right)\log(0.16)$

$= \log_{5/2}(1/2)\log(2/5)^2 = \log 4$

$\Rightarrow$ x = 4

(D) $3^x \dfrac{(1/3)}{1-1/3} = \dfrac{2(5^2)}{1-1/5}$

$\Rightarrow \dfrac{1}{2}(3^x) = \dfrac{1}{2}(5^3) \Rightarrow$ x = 3 $\log_3 5$

60. Ans. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p) (C) $\rightarrow$ (s), (D) $\rightarrow$ (q)

(A) $\dfrac{ydx - xdy}{y^2} = dx + \dfrac{dy}{y^2} \Rightarrow d\left(\dfrac{x}{y}\right) = dx + \dfrac{dy}{y^2} \Rightarrow \dfrac{x}{y} = x - \dfrac{1}{y} + k$

$\Rightarrow$ x = xy – 1 + ky $\Rightarrow$ (x + 1) (1 – y) = cy

(B) $(2x - 10y^3)\dfrac{dy}{dx} + y = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{10y^3 - 2x} \Rightarrow \dfrac{dx}{dy} = \dfrac{10y^3 - 2x}{y}$

$\dfrac{dx}{dy} = 10y^2 - 2\dfrac{x}{y} \Rightarrow \dfrac{dx}{dy} + \dfrac{2}{y}x = 10y^2 \Rightarrow xy^2 = 10\dfrac{y^5}{5} + c$

$\Rightarrow xy^2 = 2y^5 + C$

(C) $\sec^2 y \dfrac{dy}{dx} + \tan y = 1$ put tan y = t, $\sec^2 y \dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\Rightarrow \dfrac{dt}{dx} = 1 - t \Rightarrow \ln(1 - t) = -cx \Rightarrow 1 - t = e^{-cx} \Rightarrow t = 1 - e^{-cx}$

$= \tan y = 1 + ce^{-x}$

(D) $\sin y \dfrac{dy}{dx} = \cos y(1 - x\cos y)$

put cos y = t, $\sin y \dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\Rightarrow \dfrac{dt}{dx} = t(1 - tx)$

sec y = x +1 + ce^x