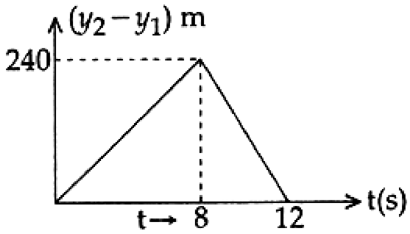
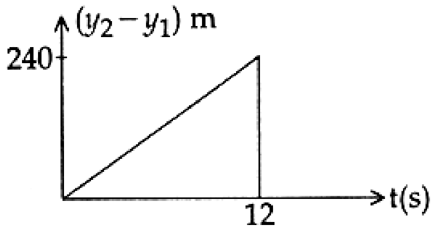
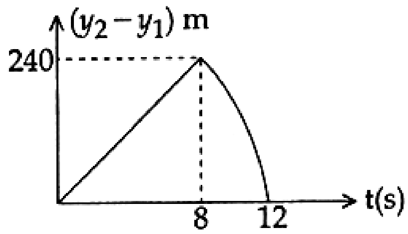
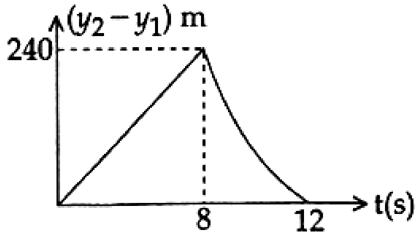
PART A (PHYSICS)

1. Two stones are thrown up simultaneously from the edge of cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stone do not rebound after hitting the ground and neglect air resistance, take g = 10m/s^2)
(The figures are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

Sol. (3)
Time of flight of the two particles will be 8s and 12s.
Till both particles are in air, separation between them will be increase linearly (As relative acceleration is zero)
When one particle (having time of flight 8s) will hit ground it will remain stationary and the separation will decreases such that magnitude of slope continuously increases (as relative speed is increasing)

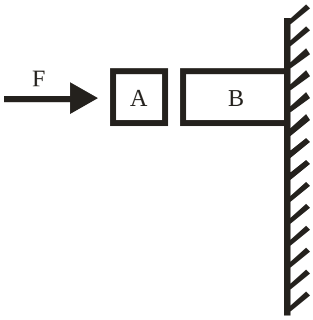
2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is:
(1) 2% (2) 3 % (3) 1% (4) 5%

Sol. (2)
$$g = 4\pi^2 \frac{L}{T^2}$$
$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$
Here, $\frac{\Delta L}{L} = \frac{1}{200}$ and $\frac{\Delta T}{T} = \frac{1}{90}$

Thus, $\frac{\Delta g}{g} = 0.005 + 0.022 = 0.027 \sim 3\%$

So most appropriate option is 3%

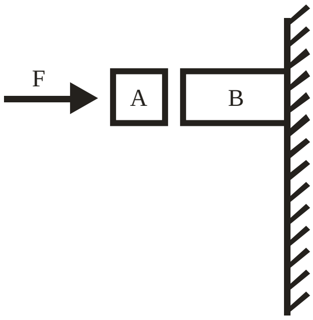
3. Given the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



(1) 100 N (2) 80 N (3) 120 N (4) 150 N

Sol. (3)

Assuming the force is F is sufficient enough to generate normal force at wall to avoid relative slipping.

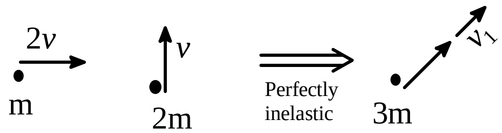


Taking both blocks as single system, only vertical force acting on system is friction at wall, which has to balance total weight. So friction is 120N.

4. A particle of mass m moving in the *x* direction with speed 2*v* is hit by another particle of mass 2m moving in the *y* direction with speed *v*. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:

(1) 44 % (2) 50 % (3) 56 % (4) 62 %

Sol. (3)



Conservation of momentum

$$E_i = \frac{1}{2}m(2v)^2 + \frac{1}{2}2mv^2$$

$$= 3\ mv^2$$

$$E = \frac{1}{2}\times 3m \times \frac{8}{3\times 3}v^2$$

$$= \frac{4mv^2}{3}$$

$$3m\overrightarrow{v_1} = 2mv\hat{i} + 2mv\hat{j}$$

$$\vec{v_1} = \frac{2v}{3}\left[\hat{i} + \hat{j}\right]$$

$$\left|\vec{v_1}\right| = \frac{2\sqrt{2}v}{3}$$

$$\text{Loss in } E = \frac{E_i - E_f}{E_i} \times 100$$

$$= \frac{\left(3 - \frac{4}{3}\right)}{3} \times 100$$

$$= \frac{(9-4)}{9} \times 100$$

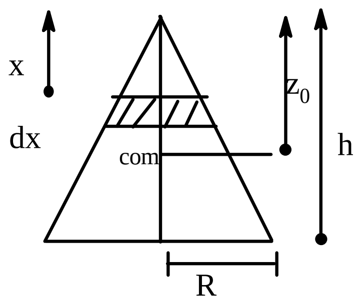
$$= \frac{5 \times 100}{9}$$

$$\simeq 56\%$$

5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to:

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$ (3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

Sol. (2)



$$\frac{x}{r} = \frac{h}{R}$$

$$r = \frac{xR}{h}$$

$$z_0 = \frac{\int x\,dm}{\int dm}$$

dm: disc of 'dx' thickness

$$z_0 = \frac{\int x\pi\left(\frac{xR}{h}\right)^2 \rho\,dx}{\int \rho\pi\left(\frac{xR}{h}\right)^2 dx}$$

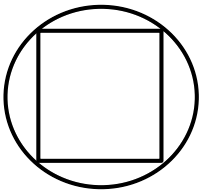
$$= \frac{\int_0^h x^3 dx}{\int_0^h x^2 dx} = \frac{\frac{h^4}{4}}{\frac{h^3}{3}}$$

$$= \frac{3h}{4}$$

6. From a solid sphere of mass M and radius R a cube of maximum possible value is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is:

(1) $\frac{MR^2}{32\sqrt{2}\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\pi}$ (3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\pi}$

Sol. (3)



Cube body diagonal is diameter of sphere

$\sqrt{3}a = 2R$

$$\rho = \frac{m}{\frac{4}{3}\pi R^3}$$

Mass of cube $m = \rho . a^3$

$$= \frac{m}{\frac{4}{3}\pi R^3} \times \left(\frac{2R}{\sqrt{3}}\right)^3$$

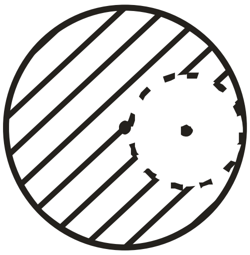
$$= \frac{2m}{\sqrt{3}\pi}$$

$$I = \frac{ma^2}{6}$$

$$= \frac{\frac{2m}{\sqrt{3}\pi} \times \left(\frac{2R}{\sqrt{3}}\right)^2}{6} = \frac{4mR^2}{9\sqrt{3}\pi}$$

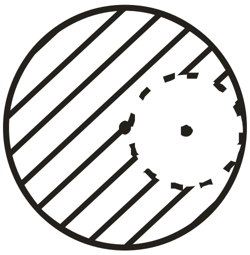
7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of cavity thus formed is

(G = gravitation constant)



(1) $\frac{-Gm}{2R}$ (2) $\frac{-Gm}{R}$ (3) $\frac{-2Gm}{3R}$ (4) $\frac{-2Gm}{R}$

Sol. (2)



m' = mass of cavity

$$m' = \frac{m}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3$$

$$= \frac{m}{8}$$

For the complete solid sphere

$$V_{solid} = \frac{-Gm}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)$$

$$= \frac{-Gm}{2R^3}\left[3R^2 - \frac{R^2}{4}\right]$$

$$= \frac{-Gm}{2R}\frac{11}{4}$$

$$= \frac{-11Gm}{8R}$$

$$V_{cavity} = \frac{-3Gm'}{2(R/2)} = \frac{-3Gm}{8R}$$

$$V_{Rest} = V_{Solid} - V_{Cavity} = V_{Rest} = -\frac{Gm}{R}$$

8. A pendulum made of a uniform wire of cross sectional area A has time period. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:

(g = gravitational acceleration)

(1) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (2) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{1}$

(3) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (4) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

Sol. (1)

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \dots (1)$$

When the body is attached with another mass m, then

$$\frac{Mg}{A} = Y \times \frac{\Delta L}{L}$$

$$\Delta L = \frac{MgL}{AY}$$

Hence $T_m = 2\pi\sqrt{\frac{L+\Delta L}{g}}$

$$T_m = 2\pi\sqrt{\frac{L+\frac{MgL}{AY}}{g}} \quad \dots (2)$$

Dividing (2) by (1)

$$\left(\frac{T_m}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\frac{1}{Y} = \left(\frac{T^2{}_m}{T^2} - 1\right)\frac{A}{Mg}$$

9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

(1) $T \propto e^{-R}$ (2) $T \propto e^{-3R}$ (3) $T \propto \frac{1}{R}$ (4) $T \propto \frac{1}{R^3}$

Sol. (3)

$$\frac{U}{V} \propto T^4$$

$$U = T^4 V \quad \dots (1)$$

$$V = \frac{4}{3}\pi R^3 \text{ and } dV = 4\pi R^2 dR$$

Differentiating (1)

$$dU = V4T^3\, dT + T^4 dV \quad \dots (2)$$

$$dW = PdV = P4\pi R^2 dR$$

And as the process is adiabatic

dW + dU = 0

$$P4\pi R^2 dR + \frac{4}{3}\pi R^3 4T^3 dT + T^4 4\pi R^2 dR = 0$$

$$P = \frac{4}{3V} = \frac{T^4 V}{3V} = \frac{T^4}{3}$$

$$\therefore \quad \frac{T^4}{3} 4\pi R^2 dR + \frac{4}{3}\pi R^3 dT + 4\pi R^2 T^4 dR = 0$$

$$\frac{TdR}{3} + \frac{4RdT}{3} + TdR = 0$$

$$\frac{4TdR}{3} + \frac{4RdT}{3} = 0$$

$$\int \frac{dR}{R} + \int \frac{dT}{T} = 0$$

$$\ell nR + \ell nT = C$$

RT = C

$$T \propto \left(\frac{1}{R}\right)$$

10. S solid body of constant heat capacity 1 J/^{0}C is being heated by keeping it in contact with reservoirs in two ways:
(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.
In both the cases body is brought from initial temperature 100^{0}C to final temperature 200^{0}C. Entropy change of the body in the two cases respectively is:
(1) *l*n2, 4*l*n2 (2) *l*n2, *l*n2 (3) *l*n2, 2*l*n2 (4) 2*l*n2, 8*l*n2

Sol. (2)

$$ds = \frac{dq}{T}$$

$$\int ds = \int_{T_1}^{T_2} \frac{CdT}{T}$$

$$\Delta S = C \ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S_1 = C \ln 2$$

For 2nd process temperature change is same thus change in entropy will be same.

$$\Delta S_2 = C \ln 2 = \Delta S_1$$

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes and adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The volume of the gas. The value of q is: $\left(\gamma = \frac{C_P}{C_V}\right)$
(1) $\frac{3\gamma+5}{6}$ (2) $\frac{3\gamma-5}{6}$ (3) $\frac{\gamma+1}{2}$ (4) $\frac{\gamma-1}{2}$

Sol. (3)

$TV^{\gamma-1} = \text{const} \Rightarrow T \propto V^{(1-\gamma)}$

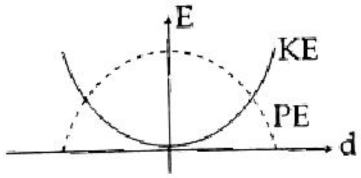
$\Rightarrow V_{rms} \propto \sqrt{T} \propto V^{\left(\frac{1-\gamma}{2}\right)}$

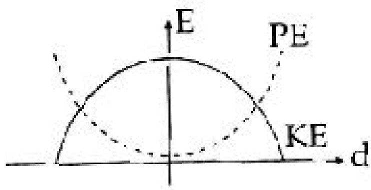
$\therefore \quad t_{collision} \propto \frac{\bar{\ell}}{v_{rms}} \propto \frac{(v)}{v^{(1-\gamma)/2}} \propto v^{1-\frac{1-\gamma}{2}} \propto v^{\frac{1+\gamma}{2}}$

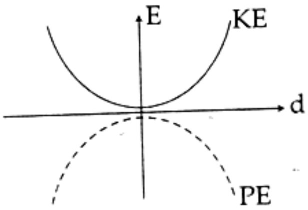
[Using ℓ = mean free path $= \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2} \propto V$,

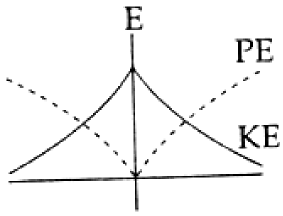
where d = molecule diameter
N = total no. of molecules]

12. For a simple pendulums, a graph is plotted between its kinetic energy KE and potential energy (PE) against its displacement d. Which one of the following represents these correctly?
(graphs are schematic and not drawn to scale).

(1) 

(2) 

(3) 

(4) 

Sol. (2)

Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

Potential energy $= \frac{1}{2}m\omega^2x^2$

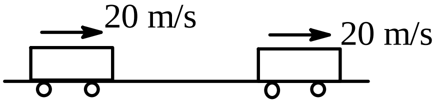
Total energy = $KE + PE$

$= \frac{1}{2}m\omega^2A^2 = \text{constant}$

13. A train is moving on a straight track with speed $20 ms^{-1}$. It is blowing its whistle at the frequency of $1000 Hz$. The percentage change in the frequency heard by a person standing near the track as the train passes him is $\left(\text{Speed of sound} = 320\,ms^{-1}\right)$ close to:

(1) 6% (2) 12% (3) 18% (4) 24%

Sol. (2)



$f'_{app} = f_0\left(\frac{v}{v-u_s}\right) \qquad f'_{rec} = f_0 = \left(\frac{v}{v+u_s}\right)$

$= f_0\left[\dfrac{320}{320-20}\right]$ $\qquad f'_{rec} = f_0 \times \dfrac{320}{320+20}$

$= f_0 \times \dfrac{320}{300}$ $\qquad = f_0 \times \dfrac{320}{340}$

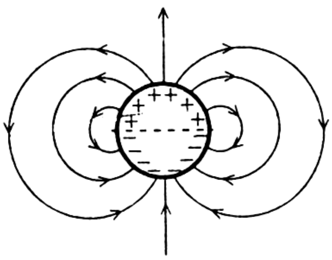
$= f_0 \times \dfrac{16}{15}$ $\qquad = f_0 \times \dfrac{16}{17}$

Change in frequency = $f_0\left(\dfrac{16}{15} - \dfrac{16}{17}\right)$

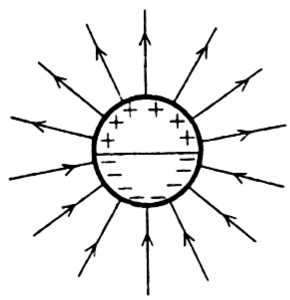
$= f_0 \times 0.125$

% change = $\dfrac{f_0 \times 0.125}{f_0} \times 100 = 12.5y. \simeq 12y$

14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in: (figures are schematic and not drawn to scale).



(1)



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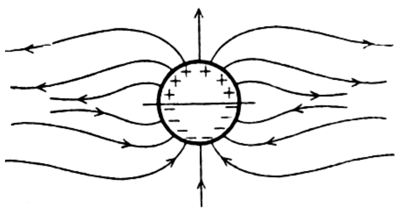

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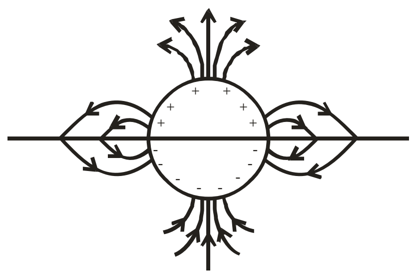


(4)

Sol. (1)

15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equi potential surfaces with potentials $\dfrac{3V_0}{2}, \dfrac{5V_0}{4}, \dfrac{3V_0}{4}$ and $\dfrac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$ (2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$ (4) $2R < R_4$

Sol. (3, 4)

$$\frac{KQ\left(3R^2 - R_1^2\right)}{2R^3} = \frac{3}{2}\frac{KQ}{R}$$

$$\boxed{R_1 = 0}$$

$$\frac{KQ\left(3R^2 - R_2^2\right)}{2R^3} = \frac{5}{4}\frac{KQ}{R}$$

$$6R^2 - 2R_2^2 = 5R^2$$

$$\boxed{R_2 = \frac{1}{\sqrt{2}}R = 0.707R}$$

$$\frac{KQ}{R_3} = \frac{3}{4}\frac{KQ}{R}$$

$$\boxed{R_3 = \frac{4}{3}R = 1.33R}$$

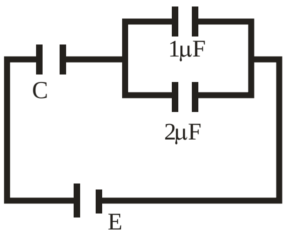
$$\frac{KQ}{R_4} = \frac{1}{4}\times\frac{KQ}{R}$$

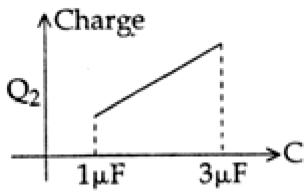
$$\boxed{R_4 = 4R}$$

$$R_2 < R_4 - R_3 \Rightarrow 0.707R < 4R - 1.33R$$

$$2R < R_4 \Rightarrow 2R < 4R$$

16. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varies from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by : (figure are drawn schematically and are not to scale).



(1)


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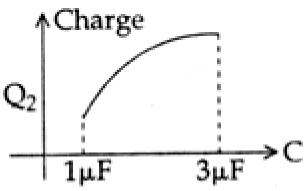

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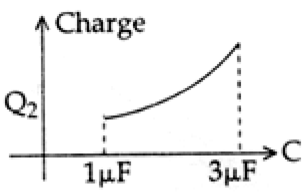

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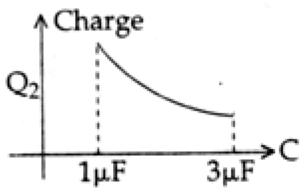


Sol. (2)

$$\Rightarrow \frac{Q_2}{2} = \frac{Q_1}{1}$$

$$\Rightarrow Q_1 + Q_2 = Q$$

$$\frac{Q_2}{2} + Q_2 = Q$$

$$Q_2 = \frac{2Q}{3}$$

$Q = \dfrac{3CE}{3+C}$

$Q_2 = \dfrac{2}{3} \times \dfrac{3CE}{3+C}$

$= 2E\left(\dfrac{C+3}{C+3} - \dfrac{3}{C+3}\right)$

$= 2E\left(1 - \dfrac{3}{C+3}\right)$

$\dfrac{dQ_2}{dC} = \dfrac{2E \times 3}{(C+3)^2}$

$C \to \infty, \dfrac{dQ_2}{dC} \to 0$

17. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is $2.5\times10^{-4}\,ms^{-1}$. If the electron density in the wire is $8\times10^{28}\,m^{-3}$, the resistivity of the material is close to:

(1) $1.6\times10^{-8}\,\Omega m$ (2) $1.6\times10^{-7}\,\Omega m$

(3) $1.6\times10^{-6}\,\Omega m$ (4) $1.6\times10^{-5}\,\Omega m$

Sol. (4)

V = 5V

l = 0.1 m

$v_d = 2.5\times10^{-4}\ ms^{-1}$

$I = neAv_d$

$\dfrac{V}{R} = neAv_d$

$\dfrac{V\times A}{\rho\ell} = neAv_d$

$\rho = \dfrac{V}{ne\ell v_d}$

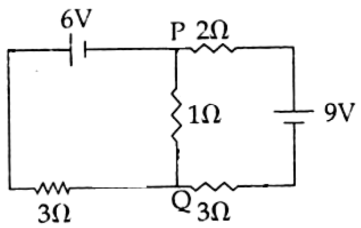
$= \dfrac{5}{8\times10^{28}\times1.6\times10^{-19}\times0.1\times2.5\times10^{-4}}$

$\rho = 1.6\times10^{-5}\ \Omega m$

18. In the circuit shown, the current in the 1Ω resistor is:

(1) 1.3 A, from P to Q (2) 0 A

(3) 0.13 A, from Q to P (4) 0.13 A, from P to Q



Sol. (3)

From P to Q

$$i = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{1 + R\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

$$= \frac{\frac{9}{5} - \frac{6}{3}}{1 + \left(\frac{1}{3} + \frac{1}{5}\right)} = \frac{\frac{9}{5} - 2}{1 + \frac{8}{15}}$$

$$= -\frac{\frac{1}{5}}{23/15} = -\frac{3}{23} = -0.13A$$

Current is flowing from Q to P = 0.13A

19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then:

(1) $\vec{F}_1 = \vec{F}_2 = 0$

(2) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards

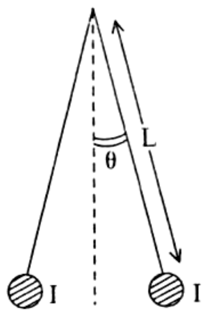
(3) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$

(4) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

Sol. (1)

Magnetic field due to inner solenoid at the location of outer solenoid will be zero. So force on inner solenoid will be symmetrical.

20. Two long current carrying thin wires, both with current I, are held by insulating thread of length L and are in equilibrium as shown in the figure, with threads making and angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is: $(g = \text{gravitational acceleration})$



(1) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

(2) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

(3) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$

(4) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

Sol. (2)

$$T\sin\theta = \frac{\mu_0 i^2}{4\pi\ell\sin\theta}dx \qquad \ldots (1)$$

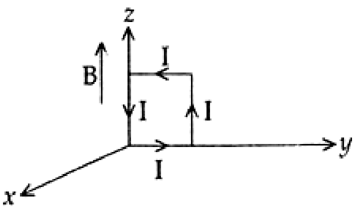
$$T\cos\theta = \lambda g dx \qquad \ldots (2)$$

$$\frac{T\sin^2\theta\ 4\pi\ell}{T\cos\theta} = \frac{\mu_0 i^2 dx}{\lambda g dx}$$

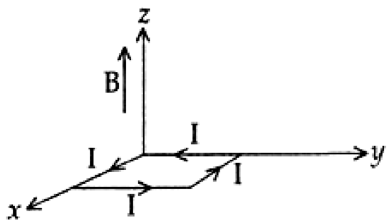
$$i^2 = \frac{4\pi\lambda g\ell\sin^2\theta}{\mu_0\cos\theta}$$

$$i = 2\sin\theta\sqrt{\frac{\pi\lambda g\ell}{\mu_0\cos\theta}}$$

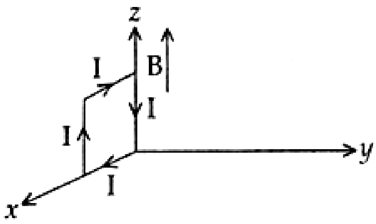
21. A rectangular loop of sides 10 cm and 5 cm carrying a current 1 of 12 A is placed in different orientations as shown in the figures below:



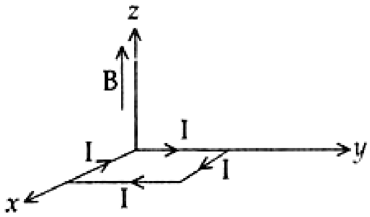
(a)



(b)



(c)



(d)

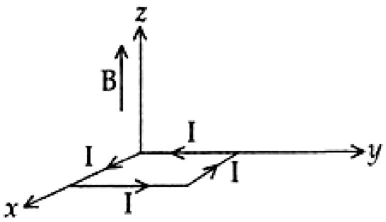
If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(1) (a) and (b), respectively (2) (a) and (c), respectively

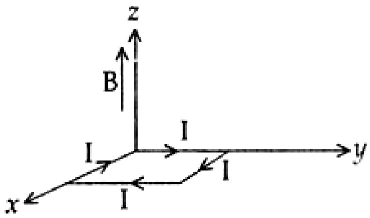
(3) (b) and (d), respectively (4) (b) and (c), respectively

Sol. (3)

Let $\vec{A}$ = Area vector of loop



(b)



(d)

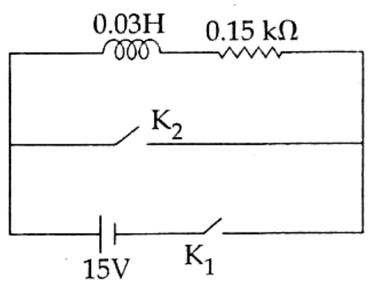
Stable Equilibrium

$\vec{A}$ ↑↑ $\vec{B}$

Unstable Equilibrium

$\vec{A}$ ↑↓ $\vec{B}$

22. An inductor $(L = 0.03H)$ and a resistor $(R = 0.15\,k\Omega)$ are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0, K_1$ is opened and key K_2 is closed simulataneously. At $t = 1ms$, the current in the circuit will be: $(e^5 \cong 150)$



(1) 100 mA (2) 67 mA (3) 6.7 mA (4) 0.67 mA

Sol. (4)

$i = 0.1A$

$i = i_0 e^{-\frac{R.t}{L}}$

$i = 0.1\, e^{\frac{-0.15}{.03}\times 10^{-3}\times 10^{3}}$

$= 0.1e^{-5}$

$= \frac{0.1}{150}A = \frac{100}{150}mA = 0.67mA$

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1m from the diode is:

(1) $1.73\,V/m$ (2) $2.45\,V/m$ (3) $5.48\,V/m$ (4) $7.75\,V/m$

Sol. (2)

$I = \frac{P}{A} = \frac{0.1}{4\pi} = \frac{1}{40\pi}$

$\frac{I}{2} = \frac{1}{2}\in_0 E_{rms}^2 C$ (Intensity is equally shared between $\vec{E} \,\&\, \vec{B}$)

$\frac{1}{80\pi} = \frac{1}{2}\in_0 E_{rms}^2 C$

$\frac{1}{40\pi} = \in_0 E_{rms}^2 C$

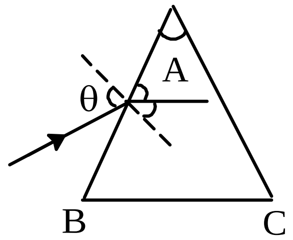
$\frac{1}{4\pi\in_0}\times\frac{1}{10} = E_{rms}^2 C$

$\frac{9\times 10^9}{10\times 3\times 10^8} = E_{rms}^2$

$E_{rms} = \sqrt{3}$

$E_0 = \sqrt{3}\times\sqrt{2} = 2.45\,v/m$

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided:



(1) $\theta > \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (2) $\theta < \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(3) $\theta > \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (4) $\theta < \sin^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Sol. (1)

$e = 90^0$

$\pi_2 = C$

$\pi_1 = A - C$

$1\sin\theta_0 = \mu\sin\pi_1 = \mu\sin(A-C)$

$\theta_0 = \sin^{-1}[\mu\sin(A-C)] = \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\frac{1}{\mu}\right)\right]$

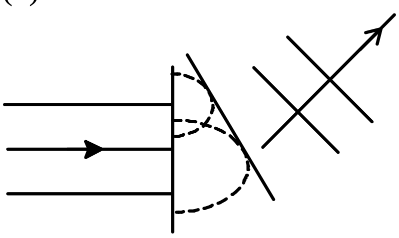
$\theta > \theta_0$

25. On a hot summer night, the refractive index of air is smallest near the ground and increased with height from the ground. When a light beam is directed horizontally, the Huygen's principle leads us to conclude that as it travels, the light beam:

(1) becomes narrower (2) goes horizontally deflection

(3) bends downwards (4) bends upwards

Sol. (4)



26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:

(1) $1\mu m$ (2) $30\mu m$ (3) $100\mu m$ (4) $300\mu m$

Sol. (2)

$\frac{x}{25cm} = 1.22\frac{\lambda}{d} = \frac{1.22\times500nm}{2\times0.25cm}$

$\Rightarrow$ x = 30 μm

27. As an electron makes a transition from an excited state to the ground state of a hydrogen- like atom/ ion:
(1) its kinetic energy increases but potential energy and total energy decrease
(2) kinetic energy, potential energy and total energy decrease
(3) kinetic energy decrease, potential energy increases but total energy remains same
(4) kinetic energy and total energy decreases but potential energy increases

Sol. (1)

$$\text{K.E.} = \frac{13.6\text{eV}}{n^2}$$

$$\text{P.E.} = \frac{-27.2\text{eV}}{n^2}$$

E = –K.E.

28. Match **List-I** (fundamental Experiment) with **List-II** (its conclusion) and select the correct option from the choices given below the list:

	List-I		List-II
(A)	Franck-Hertz Experiment	(i)	Particle nature of light
(B)	Photo-electric experiment	(ii)	Discrete energy levels of atom
(C)	Davision-Germer Experiment	(iii)	Wave nature of electron
		(iv)	Structure of atom

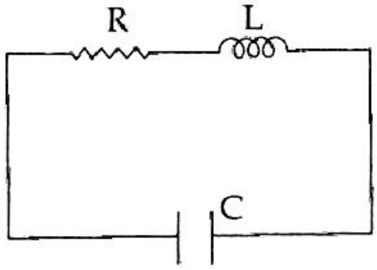
(1) $(A)-(i)\ (B)-(iv)\ (C)-(iii)$
(2) $(A)-(ii)\ (B)-(iv)\ (C)-(iii)$
(3) $(A)-(ii)\ (B)-(i)\ (C)-(iii)$
(4) $(A)-(iv)\ (B)-(iii)\ (C)-(ii)$

Sol. (3)

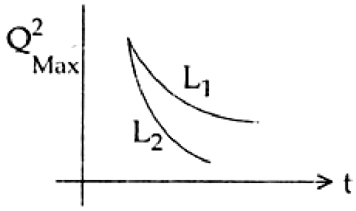
29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/ are:
(1) 2 MHs only
(2) 2005 kHz, and 1995 kHz
(3) 2005 kHz, 2000 kHz and 1995 kHz
(4) 2000 kHz and 1995 kHz

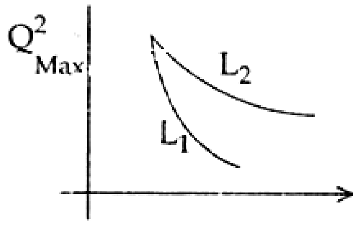
Sol. (3)
In amplitude modulation, modulated signal consists of the carrier wave of frequency 2000KHz plus two sinusoidal waves each with a frequency slightly different from original i.e., 1995 KHz and 2005 KHz.
[Refer NCERT class XII Part B Page No. 525]

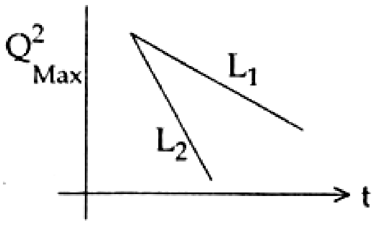
30. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

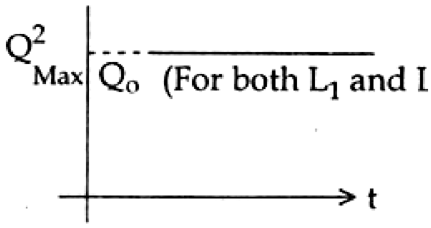


If a student plots graphs of the square of maximum charge $\left(Q_{max}^2\right)$ on the capacitor with time (t) for two different values L_1 and $L_2\left(L_1 > L_2\right)$ of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

Sol. (1)

More is the induction more is the tendency to oppose current. Just after time dt, current constituted will be more if inductance is less. So the charge that has decayed in time dt will be more if inductance is less.

PART – B (CHEMISTRY)

31. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin?

(1) $\frac{1}{103}$ (2) $\frac{1}{206}$ (3) $\frac{2}{309}$ (4) $\frac{1}{412}$

Sol. **(4)**

$2C_8H_7SO_3Na + Ca^{2+} \rightarrow \left(C_8H_7SO_3\right)_2 Ca + 2Na^+$

$2\,mol\ C_8H_7SO_3Na \equiv 1\,mol\,Ca^{2+}$

$\Rightarrow \left(2\times 206\right)g\ C_8H_7SO_3Na \equiv 1\,mol\,Ca^{2+}$

Maximum uptake of

$Ca^{2+} = \frac{n_{Ca^{2+}}}{W_{resin}\left(g\right)}$

$$= \frac{1 \text{mol}}{(2 \times 206) \text{g}}$$

$$= \frac{1}{412} \frac{\text{mol}}{\text{g}}$$

32. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of $4.29\,\overset{o}{A}$. The radius of sodium atom is approximately :

(1) $1.86\,\overset{o}{A}$ (2) $3.22\,\overset{o}{A}$ (3) $5.72\,\overset{o}{A}$ (4) $0.93\,\overset{o}{A}$

Sol. **(1)**

$$\sqrt{3}a = 4r$$

$$\Rightarrow \sqrt{3} \times 4.29 = 4 \times r$$

$$\Rightarrow r = \frac{1.732 \times 4.29}{4}$$

$$= \frac{1.7 \times 4.3}{4}$$

$$\approx 1.86$$

33. Which of the following is the energy of a possible excited state of hydrogen?

(1) $+13.6\,\text{eV}$ (2) $-6.8\,\text{eV}$ (3) $-3.4\,\text{eV}$ (4) $+6.8\,\text{eV}$

Sol. **(3)**

$$E = -13.6 \times \frac{Z^2}{n^2}$$

$$= -13.6 \times \frac{1^2}{2^2}$$

$$= -3.4\,\text{eV}$$

34. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is :

(1) ion - ion interaction (2) ion - dipole interaction
(3) London force (4) hydrogen bond

Sol. **(2)**

$$U_{\substack{\text{intermolecular} \\ \text{interaction}}} \propto \frac{1}{r^3}$$

35. The following reaction is performed at 298 K.

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The standard free energy of formation of $NO(g)$ is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_2(g)$ at 298 K? ($K_p = 1.6 \times 10^{12}$)

(1) $R(298) ln(1.6 \times 10^{12}) - 86600$ (2) $86600 + R(298) ln(1.6 \times 10^{12})$

(3) $86600 - \frac{ln(1.6 \times 10^{12})}{R(298)}$ (4) $0.5[2 \times 86,600 - R(298) ln(1.6 \times 10^{12})]$

Sol. **(4)**

$$\Delta G^0_{rkn} = -RT \ell n\, K_p$$

$$\Rightarrow 2\Delta G^0_{f_{NO_2}} - \left\{2\Delta G^0_{f_{NO}} + \Delta G^0_{f_{O_2}}\right\} = \Delta G^0_r = -RT\,\ell n\,K_p$$
$$\Rightarrow 2\Delta G^0_{f_{NO_2}} = 2\Delta G^0_{f_{NO}} - RT\ell nK_p$$
$$\Rightarrow \Delta G^o_{f_{NO_2}} = \frac{1}{2}\left\{2\times 86.6kJ - R(298)\ell n\left(1.6\times 10^{12}\right)\right\}$$
$$\Delta G^o_{f_{NO_2}} = 0.5\left\{2\times 86,600 - R(298)\ell n\left(1.6\times 10^{12}\right)\right\}$$

36. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is:

(1) 32 (2) 64 (3) 128 (4) 488

Sol. **(2)**

$$\Rightarrow \frac{P^o - P}{P^o} = \frac{n_{solute}}{n_{solvent}} = \frac{W_{solution}/M_{Solute}}{W_{Acetone}/M_{Acetone}}$$

$$\Rightarrow \frac{185-183}{185} = \frac{1.2/M_o}{100/58} \Rightarrow M_o = \frac{185}{2}\times 1.2\times\frac{58}{100}$$

≈ 64

37. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}$, $[B] = 2$ and $[C] = \frac{1}{2}$. The reaction proceeds in the : $[R = 8.314 J/K/mol,\ e = 2.718]$

(1) forward direction because $Q > K_C$ (2) reverse direction because $Q > K_C$

(3) forward direction because $Q < K_C$ (4) reverse direction because $Q < K_C$

Sol. **(2)**

$2A \to B + C,\ \Delta G^0 = 2492.2\ J/mol$

At a given time, $[A] = \frac{1}{2}$ $[B] = 2$ $[C] = \frac{1}{2}$

Reaction quotient $Q = \frac{[B]\cdot[C]}{[A]^2} = \frac{2\times\frac{1}{2}}{\left(1/2\right)^2}$

$Q = 4$

$\Delta G^0 = -RT\,\ell nK$

$\Rightarrow 2492.2 = -8.31\times 300\times \ell nK$

$\Rightarrow \ell nK = -1$

$\Rightarrow K = 1/e$

Since, $Q > K$, reaction will move in backward reaction.

38. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is : (*at. mass of Cu = 63.5 amu*)

(1) 0 g (2) 63.5 g (3) 2 g (4) 127 g

Sol. **(2)**

$Cu^{2+}_{(aq)} + 2e^- \rightarrow Cu(s)$

In the given reaction, when 2 moles of e^- are transferred, 1 mole copper will get deposited at the cathode.
So, 63.5 g of Cu is deposited.

39. Higher order (>3) reactions are rate due to :
(1) low probability of simultaneous collision of all the reaction species.
(2) increase in entropy and activation energy as more molecules are involved
(3) shifting of equilibrium towards reactants due to elastic collisions
(4) loss of active species of collision

Sol. **(1)**
Higher order (>3) reactions are rare as there is very less probability of simultaneous collision of all the reacting molecules at a particular time.

40. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is :
(1) 18 mg (2) 36 mg (3) 42 mg (4) 54 mg

Sol. **(1)**

Mill equivalence of acetic acid absorbed $= \frac{50}{1000} \times (0.06 - 0.042)$

$= 0.9$

Mass (in mg) of acetic acid absorbed $= 0.9 \times 60$

$= 54\,\text{mg}$

$\therefore$ mass (in mg) of acetic acid absorbed per gram of charcoal $= \frac{54}{3}\,\text{mg} = 18\,\text{mg}$

41. The ionic radii (in $\overset{o}{A}$) of N^{3-}, O^{2-} and F^- are respectively :
(1) 1.36, 1.40 and 1.71 (2) 1.36, 1.71 and 1.40
(3) 1.71, 1.40 and 1.36 (4) 1.71, 1.36 and 1.40

Sol. **(3)**
N^{3-}, O^{2-} and F^- are isoelectronic species; then the one with lower number of proton will have higher radius.
Hence radius: $N^{3-} > O^{2-} > F^-$
By comparing answer is (3).
1.71, 1.40 and 1.36.

42. In the context of the Hall-Heroult process for the extraction of A*l*, which of the following statements is **false**?
(1) CO and CO_2 are produced in this process
(2) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity
(3) Al^{3+} is reduced at the cathode to form A*l*
(4) Na_3AlF_6 serves as the electrolyte

Sol. **(4)** is incorrect.
In Hall-heroult process:

$Na_3AlF_6 \rightleftharpoons 3NaF + AlF_3$

$AlF_3 \rightleftharpoons Al^{3+} + 3F^-$

At anode: $F^- \rightarrow F + e^-$

At cathode: $Al^{3+} + 3e^- \rightarrow Al$

$2Al_2O_3 + 12F \rightarrow 4AlF_3 + 3O_2$

$2C + O_2 \rightarrow 2Co$

$2Co + O_2 \rightarrow 2Co_2$

The addition of Na_3AlF_3 and CaF_2 makes alumina a good conductor of electricity and lowers its fusion temperature, hence1, 2 & 3 are correct.

43. From the following statements regarding H_2O_2, choose the incorrect statement:

(1) It can act only as an oxidizing agent

(2) It decompose on exposure to light

(3) It has to be stored in plastic or wax lined glass bottles in dark

(4) It has to be kept away from dust

Sol. **(1)**

H_2O_2 can act both as oxidising as well as reducing agent.

44. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?

(1) $CaSO_4$ (2) $BeSO_4$ (3) $BaSO_4$ (4) $SrSO_4$

Sol. **(2)**

Solubility of sulphates of Group 2 decrease down the group.

$BeSO_4$ is most soluble, hence its hydration enthalpy is greater than its lattice enthalpy

45. Which among the following is the most reactive?

(1) Cl_2 (2) Br_2 (3) I_2 (4) ICl

Sol. **(4)**

$I-Cl$ is polar and hence, most reactive

46. Match the catalysts to the correct processes:

Catalyst	**Process**
(A) $TiCl_3$	(i) Wacker process
(B) $PdCl_2$	(ii) Ziegler – Natta polymerization
(C) $CuCl_2$	(iii) Contact process
(D) V_2O_5	(iv) Deacon’s process

(1) $(A)-(iii),(B)-(ii),(C)-(iv),(D)-(i)$

(2) $(A)-(ii),(B)-(i),(C)-(iv),(D)-(iii)$

(3) $(A)-(ii),(B)-(iii),(C)-(iv),(D)-(i)$

(4) $(A)-(iii),(B)-(i),(C)-(ii),(D)-(iv)$

Sol. **(2)**

47. Which one has the highest boiling point?

(1) He (2) Ne (3) Kr (4) Xe

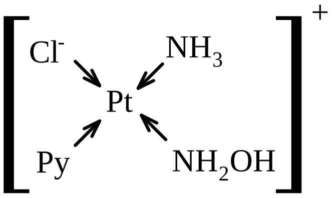
Sol. **(4)**

Molecular weight $\uparrow \Rightarrow$ B.P. $\uparrow$

48. The number of geometric isomers that can exist for square planar $\left[Pt(Cl)(py)(NH_3)(NH_2OH)\right]^+$ is (py = pyridine):

(1) 2 (2) 3 (3) 4 (4) 6

Sol. **(2)**



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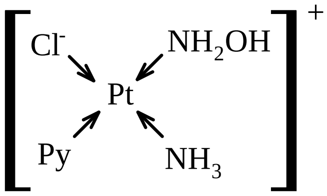
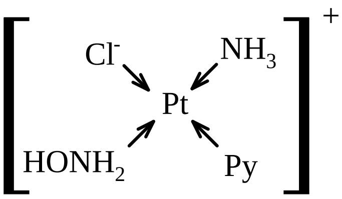


cis



cis

49. The color of $KMnO_4$ is due to:

(1) $M \rightarrow L$ charge transfer transition

(2) $d-d$ transition

(3) $L \rightarrow M$ Charge transfer transition

(4) $\sigma-\sigma^*$ transition

Sol. **(3)**

$KMnO_4$ has $Mn^{7+}; [Ar]3d^0 4s^0$

$\Rightarrow d-d$ transition not possible

It is $L \rightarrow M$ charge transfer transition.

50. **Assertion:** Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason: The reaction between nitrogen and oxygen requires high temperature.

(1) Both assertion and reason are correct, and the reason is the correct explanation for the assertion

(2) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion

(3) The assertion is incorrect, but the reason is correct

(4) Both the assertion and reason are incorrect

Sol. **(1)**

Both assertion and reason are correct statements and assertion is correctly explained by the reason.

51. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is:
(at.mas $Ag = 108; Br = 80$)

(1) 24 (2) 36 (3) 48 (4) 60

Sol. **(1)**

Mass of Br in $AgBr = \frac{141}{188} \times 80\,mg = 60\,mg$

$\therefore$ Mass of Br in organic compound $= 60\,mg$

% of Br in organic Compound $= \frac{60}{250} \times 100$

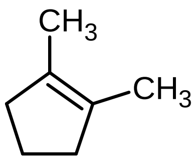
$= 24\%$

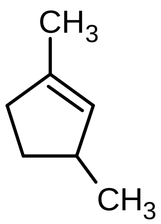
52. Which of the following compounds will exhibit geometrical isomerism?

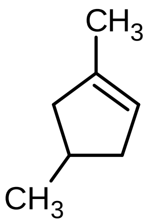
(1) 1 – Phenyl – 2 – butene (2) 3 – Phenyl – 1 – butene
(3) 2 – Phenyl – 1 butene (4) 1, 1 – Diphenyl – 1 – propene

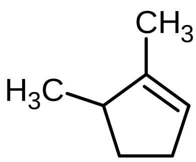
Sol. **(1)**

53. Which compound would given 5 – keto – 2 – methyl hexanal upon ozonolysis?

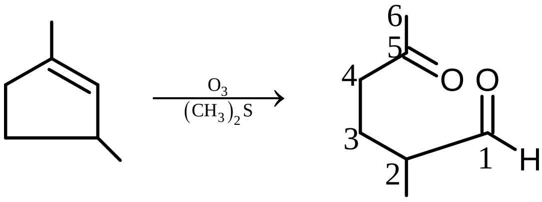
(1) 

(2) 

(3) 

(4) 

Sol. **(2)**



54. The synthesis of alkyl fluorides is best accomplished by:

(1) Free radical fluorination (2) Sandmeyer's reaction
(3) Finkelstein reaction (4) Swarts reaction

Sol. **(4)**

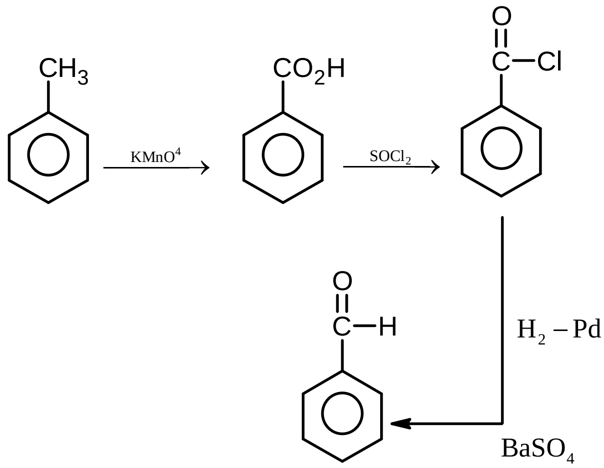
$$R - Cl \xrightarrow{AgF} R - F + AgCl$$

55. In the following sequence of reactions:

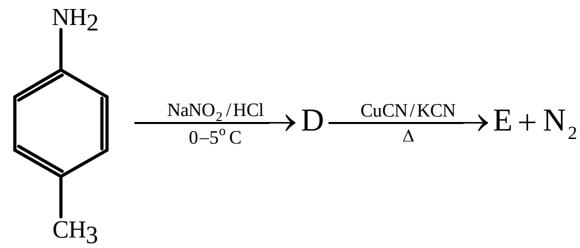
$\text{Toluene} \xrightarrow{KMnO_4} A \xrightarrow{SOCl_2} B \xrightarrow[BaSO_4]{H_2/Pd} C$, the product C is :

(1) C_5H_5COOH (2) $C_6H_5CH_3$ (3) $C_6H_5CH_2OH$ (4) C_6H_5CHO

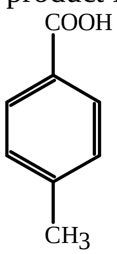
Sol. **(4)**

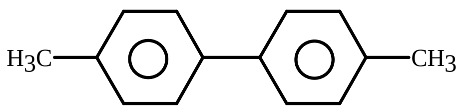


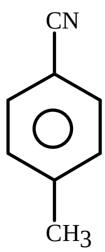
56. In the reaction

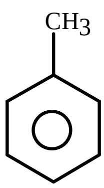


The product E is :

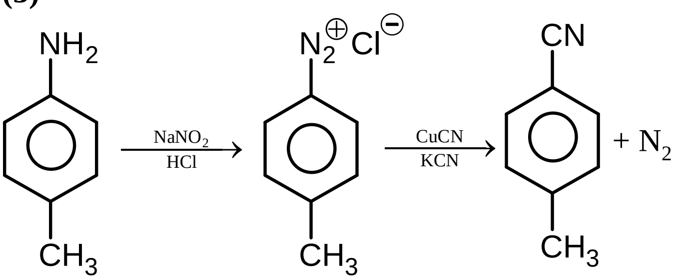
(1) 

(2) 

(3) 

(4) 

Sol. **(3)**



57. Which polymer is used in the manufacture of paints and lacquers?
(1) Bakelite (2) Glyptal
(3) Polypropene (4) Poly vinyl chloride
Sol. **(2)**
Glyptal

58. Which of the vitamins given below is water soluble?
(1) Vitamin C (2) Vitamin D (3) Vitamin E (4) Vitamin K
Sol. **(1)**
Vitamin C

59. Which of the following compounds is **not** antacid?
(1) Aluminium hydroxide (2) Cimetidine
(3) Phenelzine (4) Ranitidine
Sol. **(3)**
phenelzine

60. Which of the following compounds is **not** coloured yellow?
(1) $Zn_2\left[Fe(CN)_6\right]$ (2) $K_3\left[Co(NO_2)_6\right]$
(3) $(NH_4)_3\left[As(Mo_3O_{10})_4\right]$ (4) $BaCrO_4$
Sol. **(1)**
$Zn_2\left[Fe(CN)_6\right]$ is colourless.

PART – C (MATHEMATICS)

61. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A\times B$, each having at least three elements is:
(1) 219 (2) 256 (3) 275 (4) 510
Sol: (1)
$n(A\times B)=4\times 2=8$ atleast
$\Rightarrow$ no of subsets having 3 $={}^8C_3+{}^8C_4-{}^8C_8 \;=219$

62. A complex number z is said to be unimodular if $|z|=1$. Suppose z_1 and z_2 are complex numbers such that $\dfrac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a:
(1) straight line parallel to $x-$ axis (2) straight line parallel to $y-$axis
(3) circle of radius 2 (4) circle of radius $\sqrt{2}$
Sol: (3)
$|z_1-2z_2|^2=|2-z_1.\bar{z}_2|^2$
$\Rightarrow |z_1|^2+4|z_2|^2-2\left(z_1\bar{z}_2+\bar{z}_1.z_2\right)=4+|z_1|^2.|z_2|^2-2\left(\bar{z}_1.z_2+z_1\bar{z}_2\right)$
$\Rightarrow |z_1|=2$

63. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to:

(1) 6 (2) -6 (3) 3 (4) -3

Sol: (3)

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{\left(\alpha^{10} - \beta^{10}\right) - 2\left(\alpha^8 - \beta^8\right)}{2\left(\alpha^9 - \beta^9\right)}$$

We know, $\alpha^2 - 2 = 6\alpha$ & $\beta^2 - 2 = 6\beta$

Expression, $\frac{\alpha^8\left(\alpha^2 - 2\right) - \beta^8\left(\beta^2 - 2\right)}{2\left(\alpha^9 - \beta^9\right)}$

$$\frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{2\left(\alpha^9 - \beta^9\right)}$$

$$\frac{6}{2} = 3$$

64. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 91$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to:

(1) $(2, -1)$ (2) $(-2, 1)$ (3) $(2, 1)$ (4) $(-2, -1)$

Sol: (4)

$$AA^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & \text{.........} \end{bmatrix} = a\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow a + 2b = -4$

$2a - 2b = -2$

Solve $(a, b) = (-2, -1)$

65. The set of all values of λ for which the system of linear equations:

$2x^1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 \quad = \lambda x_3$ has a non-trivial solution,

(1) is an empty set (2) is a singleton

(3) contains two elements (4) contains more than two elements

Sol: (3)

$$\Delta = 0 \Rightarrow \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(\lambda+3) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1, -3$$

66. The number of integers greater than 6,000 that can be formed, using the digits 3,5,6,7 and 8, without repetition , is:

(1) 216 (2) 192 (3) 120 (4) 72

Sol: (2)

> 6000

$4 - \text{digit} = 3 \times {}^4C_3 \times 3!$

$4 - \text{ digit} = 5!$

192

67. The sum of coefficient of integral powers of x in the binomial expansion of $\left(1-\sqrt{2x}\right)^{50}$ is:

(1) $\frac{1}{2}\left(3^{50}+1\right)$ (2) $\frac{1}{2}\left(3^{50}\right)$ (3) $\frac{1}{2}\left(3^{50}-1\right)$ (4) $\frac{1}{2}\left(2^{50}+1\right)$

Sol: (1)

$\left(1-2\sqrt{x}\right)^{50}$

$3^{50} = {}^{50}C_0 + {}^{50}C_1.2 + {}^{50}C_2.2^2 + ...$

$1 = {}^{50}C_0 - {}^{50}C_1.2 + {}^{50}C_2.2^2 + ...$

$\therefore {}^{50}C_0 + {}^{50}C_2.2^2 + {}^{50}C_4.2^4 .... 3^{50} + 1$

68. If m is the A.M. of two distinct real numbers l and $n\left(l, n > 1\right)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

(1) $4l^2mn$ (2) $4lm^2n$ (3) $4lmn^2$ (4) $4l^2m^2n^2$

Sol: (2)

$m = \frac{\ell + n}{2}$; $n = \ell r^4$

$G_1^4 + 2G_2^4 + G_3^4 = \ell^4 r^4 \left(1 + 2r^4 + r^8\right)$

$= 4n\ell m^2$

69. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + ...$ is

(1) 71 (2) 96 (3) 142 (4) 192

Sol: (2)

$\therefore T_n = \frac{\left(n(n+1)\right)^2}{2} \Big/ \frac{n}{2}\left[2\times 1 + (n-1)^2\right]$

$= \frac{1}{4}(n+1)^2$

$\therefore S_n = \sum T_n = 96$

70. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) 4 (2) 3 (3) 2 (4) $\frac{1}{2}$

Sol: (3)

$$\operatorname*{Lt}_{\lambda\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x^2\left(\frac{\tan 4x}{4x}\right)\times 4}=2$$

71. The function,

$g(x)=\begin{cases} k\sqrt{x+1}, & 0\le x\le 3 \\ mx+2, & 3<x\le 5 \end{cases}$ is differentiable, then the value of $k+m$ is:

(1) 2 (2) $\frac{16}{5}$ (3) $\frac{10}{3}$ (4) 4

Sol: (1)

For continuity at $x=3$

$\Rightarrow 2k=2m+2$ (1)

For diff at $x=3$

$k=4m$ (2)

From (1) & (2) $\Rightarrow k=8/5, m=2/5$

72. The normal to the curve , $x^2+2xy-3y^2=0$, at $(1,1)$:

(1) does not meet the curve again
(2) meets the curve again.
(3) meets the curve again in the third quadrant.
(4) meets the curve again the fourth quadrant.

Sol: (4)

$x^2+2xy-3y^2=0$

$\Rightarrow (x+y)^2=4y^2 \Rightarrow x+y=\pm 2y$

$y=x$ $x+2y=0$

$\therefore$ Fourth quadrant

73. Let $f(x)$ be a polynomial of degree four having extreme values at $x=1$ and $x=2$. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then $f(2)$ is equal to:

(1) -8 (2) -4 (3) 0 (4) 4

Sol: (3)

$$\operatorname*{Lt}_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]$$

$$\operatorname*{Lt}_{x\to 0}=\frac{f'(x)}{2x}=\operatorname*{Lt}_{x\to 0}\frac{f''(x)}{2}$$

$\therefore f''(1)=f''(2)=0$

Also $\frac{f''(0)}{2} = 2, \text{i.e.} f''(0) = 6$

Solving we get $f(2) = 0$ s

74. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals:

(1) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (2) $\left(x^4+1\right)^{\frac{1}{4}} + c$

(3) $-\left(x^4+1\right)^{\frac{1}{4}} + c$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

Sol: (4)

$$I = \int \frac{dx}{x^2\left(x^4+1\right)^{3/4}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)}$$

Put $1+\frac{1}{x^4} = t$

$$I = -\left(\frac{x^4+1}{x^4}\right)^{1/4} + c$$

75. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log\left(36-12x+x^2\right)} dx$ is equal to

(1) 2 (2) 4 (3) 1 (4) 6

Sol: (3)

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(x-6)^2}$$

$$I = \int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2} dx$$

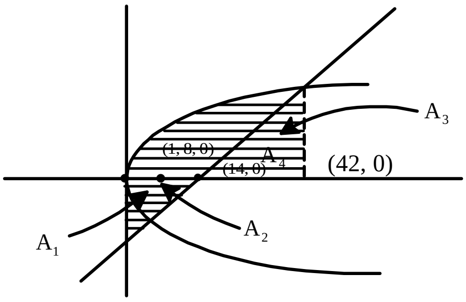
$$2I = \int_2^4 dx = 2$$

$$I = 1$$

76. The area (in sq. units) of the region described by $\left\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x-1\right\}$ is:

(1) $\frac{7}{32}$ (2) $\frac{5}{64}$ (3) $\frac{15}{64}$ (4) $\frac{9}{32}$

Sol: (4)



Required Area $= A_1 + A_2 + A_3 - A_4$

$$= \int_0^{1/8} \left|\sqrt{2}x\right|.dx + \frac{1}{2}\times\left(\frac{1}{4}-\frac{1}{8}\right)\left(\frac{1}{2}\right) + \int_0^{1/2}\left(\sqrt{2x}\right)dx - \frac{1}{2}\times 1\times\left(\frac{1}{2}-\frac{1}{4}\right) = \frac{9}{32}$$

77. Let $y(x)$ be the solution of the differential equation $(x\log x)\frac{dy}{dx} + y = 2x\log x, (x \ge 1)$. Then $y(e)$ is equal to:

(1) e (2) 0 (3) 2 (4) 2e

Sol: (3)

$$\frac{dy}{dx} + \frac{1}{2\ell n x}y = 2$$

I.F.$= \ell$ nx

$$y\,\ell nx = z\left(x\ell nx - x\right) + C$$

At $x = 1, y = 0, x = 2$

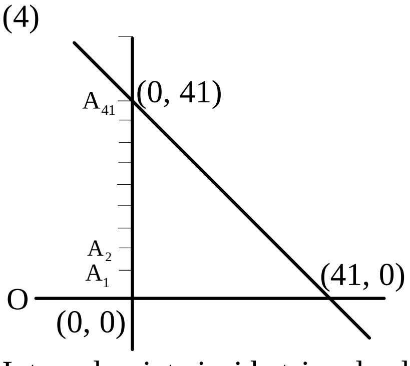
$$y = \frac{2\left(\ell ne - e\right) + 2}{\ell ne}$$

$$y = 2$$

78. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices $(0,0), (0,41)$ and $(41,0)$, is:

(1) 901 (2) 861 (3) 820 (4) 780

Sol: (4)



Integral points inside triangle along $y = 1$ are 39

$y = 2$ are 38

Total points $1 + 2.....39 = \frac{39\times 40}{2} = 780$

79. Locus of the image of the point $(2,3)$ in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0 \quad k \in R$, is a:

(1) straight line parallel to x-axis (2) straight line parallel to $y-$axis

(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

Sol: (3)

P.I of $\therefore 2x-3y+4=0$

$2-2y+3=0$

$\Rightarrow x=-1, y=2(1,2)$

$\therefore d[1,2,(2,3)]=\sqrt{2}$

$\therefore$ locus in a circle of rad $=r$

80. The number of common tangents to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$, is:

(1) 1 (2) 2 (3) 3 (4) 4

Sol: (3)

$\therefore c_1 \equiv (2,3), r_1=\sqrt{4+9+12}=5$

$c_2\ c_2 \equiv (-3,-9), r=\sqrt{9+81-26}=8$

$\therefore c_1c_2=\sqrt{25+144}=13$

$r_1+r_2=13$

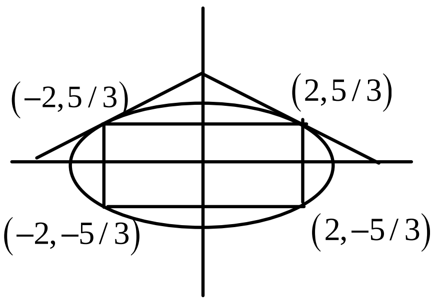
$\Rightarrow$ touching externally

81. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is:

(1) $\frac{27}{4}$ (2) 18 (3) $\frac{27}{2}$ (4) 27

Sol: (4)

$\frac{x^2}{9}+\frac{y^2}{5}=1$



$b^2/a=\frac{5}{3}$

Find P. I at these points

$A=27$

82. Let O be the vertex and Q be any point on the parabola, $x^2=8y$. If the point P divides the line segment OQ internally in the ratio $1:3$, then the locus of P is:

(1) $x^2=y$ (2) $y^2=x$ (3) $y^2=2x$ (4) $x^2=2y$

Sol: (4)

$x^2=8y$

$0=(0,0)$

Point $Q = \left(4 + \left(2t^2\right)\right)$

$P = \left(\frac{4t}{4}, \frac{2t^2}{4}\right)$

$h = t$

$h = t^2/2$

$y = \frac{x^2}{2}$

$x^2 = 2y$

83. The distance of the point $(1,0,2)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is:

(1) $2\sqrt{14}$ (2) 8 (3) $3\sqrt{21}$ (4) 13

Sol: (4)

Let Pt be $\left(\left(3\times\lambda^2\right), 4\lambda - 1, 12\lambda - 2\right)$

Lies an plane

$3\times\lambda^2 - 4\lambda + 1 + 12\lambda + 2 = 16$

$11\lambda - 1$

$\lambda = 1$

$(5,3,14)$

$\therefore (1,0,2) \,\&\, (5,3,4)$

$\sqrt{4^2 + 3^2 + 12^2} = 13$

84. The equation of the plane containing the line $2x - 5y + z = 3; x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is:

(1) $2x + 6y + 12z = 13$ (2) $x + 3y + 6z = 7$ (3) $x + 3y + 6z = 7$ (4) $2x + 6y + 12z = -13$

Sol: (3)

$P = P_1 + \lambda P_2$

$= (2+\lambda)x + (-5+\lambda)y + (1+4\lambda)z + (-3-5\lambda) = 0$

$\frac{2+\lambda}{1} = \frac{-5+\lambda}{3} = \frac{1+4\lambda}{6}$

$\lambda = -11/2$

Hence, $x + 3y + 6z = 7$

85. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ $= \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is:

(1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

Sol: (1)

$\left(\vec{a}\times\vec{b}\right)\times c = \vec{b}\left(\bar{a}.\bar{c}\right) - \bar{a}\left(\bar{b}.\bar{c}\right) = \frac{1}{3}\left|\vec{b}\right|.\left|\vec{c}\right|\left|\vec{a}\right|$

$\Rightarrow \vec{a}.\vec{c} = 0$ and $-\vec{a}\left(\vec{b}.\vec{c}\right) = -\vec{a}\left|\vec{b}\right|.\left|\vec{c}\right|\cos\theta$

$\Rightarrow \cos\theta = -1/3$

$\Rightarrow \sin\theta = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$

86. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is:

(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (2) $\frac{55}{3}\left(\frac{2}{3}\right)^{10}$ (3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

Sol: (1)

$$\frac{{}^{12}C_3 \times {}^3C_1 \times 2^9}{3^{12}}$$

87. The mean of the data set comprising of 16 observations is 16. If one of the observation values 16 is deleted and three new observation values 3,4 and 5 are added to the date, then the mean of the resultant data, is

(1) 16.8 (2) 16.0 (3) 15.8 (4) 14.0

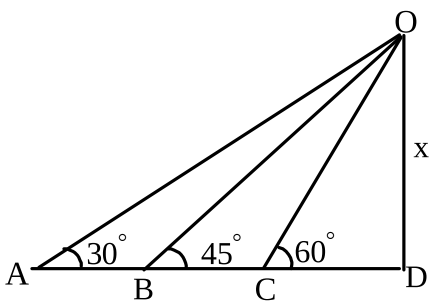
Sol: (4)

$$\mu = \frac{16\times16-16+\left(3+4+2\right)}{18} = 14.\frac{1}{3}$$

88. If the angles of elevation of the top of a tower from three collinear points A,B and C, on a line leading to the foot of the tower, are $30^\circ, 45^\circ$ and 60° respectively, then the ratio , AB: BC, is:

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1:\sqrt{3}$ (4) $2:3$

Sol: (1)



$OD = x$

$CD = x\cot 60^\circ$

$BD = x\cot 48^\circ$

$AD = x\cot 30^\circ$

$$\frac{AB}{BC} = \frac{x\left(\sqrt{3}-1\right)}{x\left(1-\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{1}$$

89. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is:

(1) $\frac{3x-x^3}{1-3x^2}$ (2) $\frac{3x+x^3}{1-3x^2}$ (3) $\frac{3x-x^3}{1+3x^2}$ (4) $\frac{3x+x^3}{1+3x^2}$

Sol: (1)

$\tan^{-1} y - \tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ solve it forms

90. The negation of $\sim s \vee (\sim r \wedge s)$ in equivalent to:

(1) $s \wedge \sim r$ (2) $s \wedge (r \wedge \sim s)$ (3) $s \vee (r \vee \sim s)$ (4) $s \wedge r$

Sol: (4)

$\sim(\sim s \vee (\sim r \wedge s)) = r \wedge .(r \vee \sim s)$

$= (s \wedge r) \vee (r \wedge \sim s)$

$= s \wedge r$