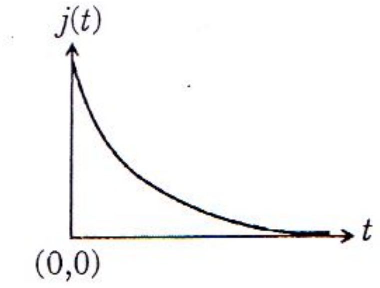
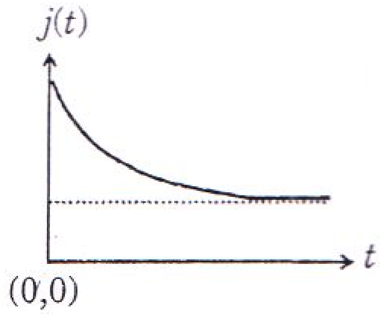
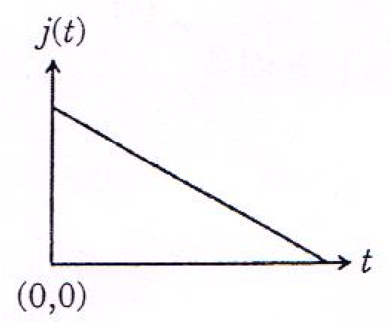
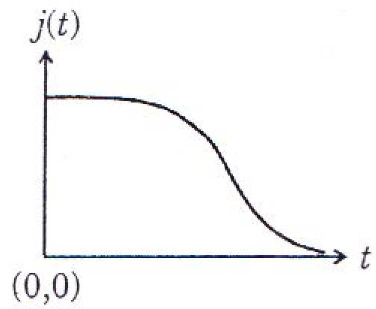
SECTION – I (PHYSICS)

1. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

(A) 

(B) 

(C) 

(D) 

Sol. **(A)**

$E = JP$

$\frac{2K\lambda}{r} = JP$

$J = \frac{2K\lambda}{Pr}$

As t $\uparrow$ J $\downarrow$ and at t = to $J \to 0$

And $\frac{dJ}{dt} \alpha \frac{dJ}{dt}$ hence option (A)

$\frac{dJ}{dt}$ is negative $\therefore \frac{dJ}{dt} < 0$

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured it a stopping potential. The relevant data for the wavelength (λ) of incident light the corresponding stopping potential (V_0) are given below :

$\lambda(\mu m)$	V_0(Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that c = 3 × 10^8 m s^{-1} and e = 1.6 x 10^{-19} C , Planck's constant (in units of J s) found from such an experiment is

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

Sol. **(B)**

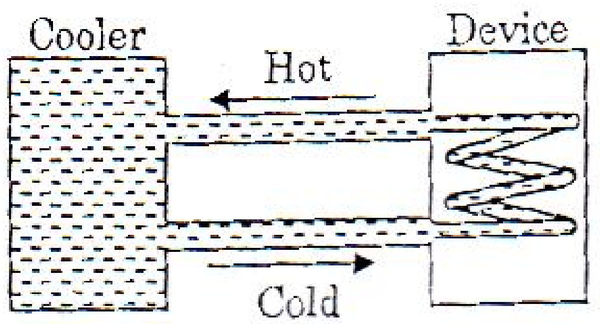
$$\frac{hc}{d}=\frac{hc}{\lambda_0}+2\times1.6\times10^{-19}$$

$$h=\frac{B\times10^{8}}{3\times10^{-7}}=\frac{hc}{\lambda_0}+1.6\times10^{-19}\times2$$

$$h=10^{15}=4\times1.6\times10^{-19}$$

$$h=6.4\times10^{-34}$$

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30 °C and the entire stored 120 litres of water is initially cooled to 10 °C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

Sol. **(B)**

heat product

$3000\ \text{j/s}\times3\times60\times60=32.4\times10^{6}\,\text{Joul}$

$120\times4.2\times10^{3}\times20=10.08\times10^{6}$ Joules. Heat extracted

$\text{P}\times3\times60\times60$

$32.4\times10^{6}-10.08\times10^{6}=\text{P}\times3\times60\times60$

4. A uniform wooden stick of mass 1.6 kg and length *l* rests is an inclined manner on a smooth, vertical wall of height h(< *l*) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio *h/l* and the frictional force *f* at the bottom of the stick are $\left(\text{g}=10\,\text{m s}^{-2}\right)$

(A) $\frac{h}{l}=\frac{\sqrt{3}}{16},\ f=\frac{16\sqrt{3}}{3}\text{N}$ (B) $\frac{h}{l}=\frac{3}{16},\ f=\frac{16\sqrt{3}}{3}\text{N}$

(C) $\frac{h}{l}=\frac{3\sqrt{3}}{16},\ f=\frac{8\sqrt{3}}{3}\text{N}$ (D) $\frac{h}{l}=\frac{3\sqrt{3}}{16},\ f=\frac{16\sqrt{3}}{3}\text{N}$

Sol. **(D)**

$$\cos30^{\circ}=\frac{\text{h}}{\text{hyp}}$$

$hup = \frac{2h}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{x}{n}$

x.

$\frac{N}{2} + N = mg$ $\qquad \frac{N\sqrt{3}}{2} = f$

$\frac{3N}{2} = mg$ $\qquad \frac{2mg}{3} \cdot \frac{\sqrt{3}}{2} = f = \frac{mg}{\sqrt{3}} = \frac{16}{\sqrt{3}}$

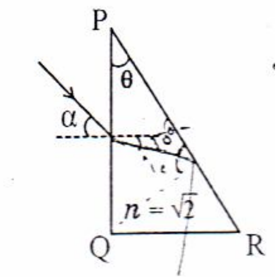
$N = \frac{2mg}{3}$

$mg\frac{\ell}{2} \cdot \frac{1}{2} = N \cdot \frac{2h}{\sqrt{3}}$

$\frac{16}{4}\ell = \frac{216}{3} \cdot \frac{2h}{\sqrt{3}}$

$\frac{h}{\ell} = \frac{3\sqrt{9}}{16}$

5. A parallel beam of light is incident from air at an angle a on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is



(A) 15^o (B) 22.5^o (C) 30^o (D) 45^o

Sol. (A)

$n = \sqrt{2}$

$\sqrt{2}\sin i = 1$

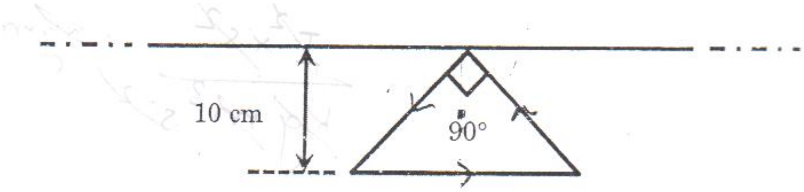
$i = 45^o$

$\sin 45^o = \sqrt{2}\sin r$

$\sin r = \frac{1}{2}$

$\theta = 15^o$

6. A conducting loop in the shape of a right angled isosceles triangle of height 10cm is kept such that the 90^o vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 As^{-1}. Which of the following statement(s) is (are) true?



(A) The induced current in the wire is in opposite direction to the current along the hypotenuse

(B) There is a repulsive force between the wire and the loop

(C) The magnitude of induced *emf* in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(D) If the loop is rotate at a constant angular speed about the wire, an additional *emf* of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.

Sol. **(B, C)**

$\phi = 2xdx\frac{\mu_o}{2\pi}\frac{i_1}{x}$

$\phi_2 = \frac{\mu 0 i_1}{\pi}\frac{(10)}{100}$

$M = \frac{\mu_o 10}{\pi \times 100}$

$\phi = Mi_2$

$\varepsilon_1 = \frac{d\phi}{dt} = M\frac{di_2}{dt} = \frac{\mu_o 10}{\pi}\frac{(10)}{100}$

$\varepsilon_1 = \frac{\mu_o}{\pi}$

7. A length-scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression (s) for l is (are) dimensionally correct?

(A) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$

(B) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(C) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$

(D) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

Sol. **(BD)**

$\frac{q^2}{\omega} = FL^2$

$K_B T = Energy = FL$

$n = L^{-3}$

(A) $\sqrt{n\frac{q^2}{\varepsilon_0}\frac{1}{K_B T}} = \sqrt{\frac{L^{-3}FL^2}{FL}} = \frac{1}{L}$

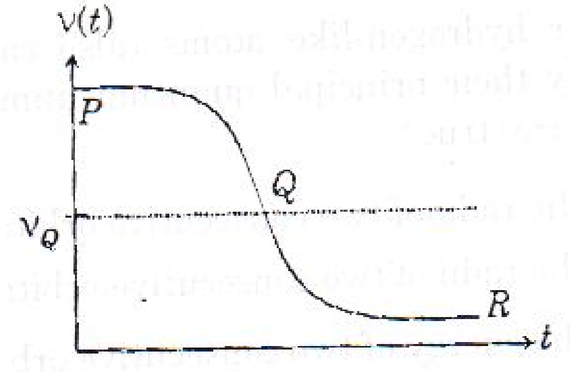
(B) $\sqrt{\frac{K_B T}{n\frac{q^2}{\varepsilon^2}}} = \sqrt{\frac{FL}{L^{-3}FL^2}} = \sqrt{L^2} = L$

(C) $\sqrt{\frac{q^2}{\varepsilon}\cdot\frac{1}{h^{2/3}}\cdot\frac{1}{K_BT}} = \sqrt{\frac{FL^2}{L^{-2}}} = \sqrt{L^3}$

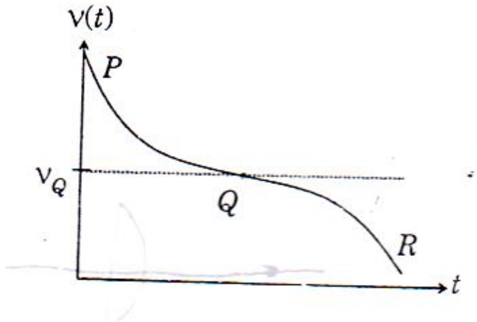
(D) $\sqrt{\frac{a^2}{\varepsilon\frac{1}{ng}}} = \sqrt{\frac{FL^2}{L^{-1}FL}} = L$

8. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line *MN* and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let v(t) represent the beat frequency measured by a person sitting in the car at time t. Let v_p , v_e and v_R be the beat frequencies measured at locations P , Q and R ,respectively. The speed of sound in air is 330 m s^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person?

(A) The plot below represents schematically the variation of beat frequency with time



(B) The plot below represents schematically the variation of beat frequency with time



(C) $v_p + v_R = 2v_Q$

(D) The rate of change in beat frequency is maximum when the car passes through Q

Sol. **(ACD)**

$$v(t) = \left(\frac{V + V_p\cos\theta}{V}\right)(121-118)$$

As $v_p\cos\theta$ decreases so *v*(t) decreases. Initial rate of decrease will be small.

$$v_P = \frac{330+\frac{50}{3}}{330}\times 3$$

$$v_Q = 3$$

$$v_R = \left(\frac{330-\frac{50}{3}}{330}\right)\times 3$$

$$v_P + v_R = 2v_Q$$

9. A piano-convex lens is made of refractive index *n*. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance 10 cm away from the lens. Which of the following statement(s) is(are) true?

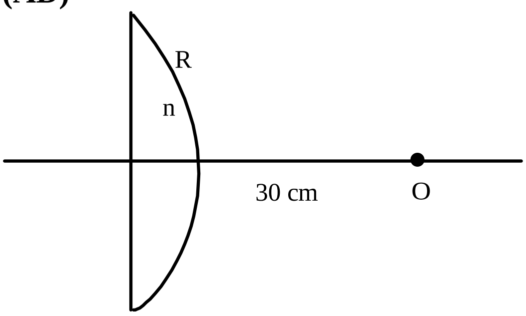
(A) The refractive index of the lens is 2.5

(B) The radius of curvature of the convex surface is 45 cm

(C) The faint images is erect and real

(D) The focal length of the lens is 20 cm

Sol. **(AD)**



$$\frac{1}{v}-\frac{1}{u}=(n-1)\left[\frac{1}{+R}-\frac{1}{\omega}\right] \qquad \frac{v}{u}=-2$$

$v = -24$

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

$$\frac{1}{+10}+\frac{1}{-30}=\frac{2}{R}$$

$$\frac{3-1}{30}=\frac{2}{R}\Rightarrow\frac{2}{30}=\frac{2}{R}\Rightarrow R=30cm$$

$$\frac{1}{v}-\frac{1}{u}=\frac{(n-1)}{30} \qquad u=-30,\ v=+60$$

$\therefore\ n = 25$

$$\frac{1}{F}=(2.5-1)\left[\frac{1}{+30}-\frac{1}{\omega}\right]$$

$$=\frac{1.5}{30}\Rightarrow F=20\,cm$$

10. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge *Ze* are defined by their principal quantum number *n*, where *n* >> 1. Which of the following statement(s) is(are) true?

(A) Relative change in the radii of two consecutive orbitals does not depend on *Z*

(B) Relative change in the radii of two consecutive orbitals varies as $1/n$

(C) Relative change in the energy of two consecutive orbitals vanes as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

Sol. **(A, B, D)**

$$r = \frac{n^2}{z}$$

$$r + \Delta r = \frac{(n+1)^2}{z}$$

$$1 + \frac{\Delta r}{r} = \frac{(n+1)^2}{n^2} \Rightarrow 1 + \frac{\Delta r}{r} = \left(1 + \frac{2}{n}\right)$$

$$\frac{\Delta r}{r} = \frac{2}{n}$$

$$mvr = \frac{nh}{2\pi}$$

$$mv(r + \Delta r) = (n+1)\frac{h}{2\pi}$$

$$\frac{r + \Delta r}{r} = \frac{n+1}{n}$$

$$1 + \frac{\Delta r}{r} = 1 + \frac{1}{n}$$

$$\frac{\Delta r}{r} = \frac{1}{n}$$

$$E = \frac{z^2}{n^2}$$

$$E + \Delta E = \frac{z^2}{(n+1)^2}$$

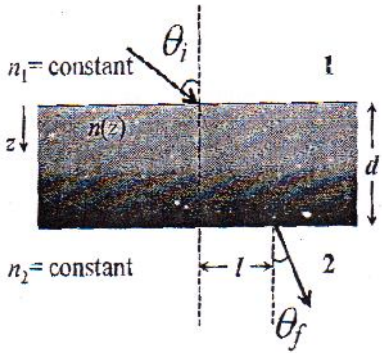
$$\frac{\Delta E}{E} + 1 = \frac{n^2}{(n+1)^2}$$

$$\frac{\Delta E}{E} + 1 = \frac{1}{\left(1 + \frac{1}{n}\right)^2}$$

$$\frac{\Delta E}{E} + \cancel{1} = \cancel{1} - \frac{2}{n}$$

$$\frac{\Delta E}{E} = -\frac{2}{n}$$

11. A transparent slab of thickness d has a refractive index $n\ (z)$ that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2\left(> n_1\right)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement(s) is(are) true?

(A) l is independent of n_2 (B) l is dependent on $n(z)$

(C) $n_1 \sin\theta_i = n_2 \sin\theta_f$ (D) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

11. **(ABC)**

12. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

12. **(D)**

Theoretical

13. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j},$$

Where $\alpha = 10/3\ ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle?

(A) The velocity $\vec{v}$ is given by $\vec{u} = (10\hat{i} + 10\hat{j})\,ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}\,N\,ms$

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\hat{k}\,N\,m$

Sol. **(ABD)**

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2 i + 2\beta t j$$

At t = 1

$$\vec{v} = 10\,i + 10\,j$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$= 0.1\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10}{3} & 5 & 0 \\ 10 & 10 & 0 \end{vmatrix}$$

$= -\frac{5}{3}\hat{k}$

$\vec{a} = \frac{d\vec{v}}{dt} = 6\alpha t\hat{i} + 2\beta\hat{j}$

At t = 1

$= 20\hat{i} + 10\hat{j}$

$\vec{F} = m\vec{a} = 2\hat{i} + \hat{j}$

$\vec{\tau} = \vec{r} \times \vec{f}$

$= \alpha\hat{k} + 2\beta\left(-\hat{k}\right) = -\frac{20}{3}\hat{k}$

14. The isotope ${}^{12}_{5}\text{B}$ having a mass 12.014 u undergoes β–decay to ${}^{12}_{6}\text{C}$. ${}^{12}_{6}\text{C}$ has an excited state of the nucleus $\left({}^{12}_{6}\text{C}^*\right)$ at 4.041 MeV above its ground state. If ${}^{12}_{5}\text{B}$ decays to ${}^{12}_{6}\text{C}^*$, the maximum kinetic energy of the β–particle in units of MeV is
(1u = 931.5 MeV/c^2, where ***c*** is the speed of light in vacuum).

Sol. **(9)**

$${}^{12}_{5}B \rightarrow \underset{\left(\substack{excited\\ state}\right)}{{}^{12}_{6}C} + {}^{0}_{-1}e + \bar{v}$$

$\left(12.014 \times 931.5\right) MeV = \left(12 \times 931.5\right) MeV + 4.041 + k$

(12.014 – 12) 931.5 = 4.041 + k

$\left(0.014\right) \times 931.5 - 4.041 = k$

13.041 – 4.041 = k

k = 9 MeV

15. Two inductors L_1 (inductance 1 mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current $\left(I_{max}/I_{min}\right)$ drawn from the battery is

Sol. **(8)**

$L_1 = 10^{-3}H$, $r_1 = 3\Omega$ $L_2 = 2 \times 10^{-3}H$, $r_2 = 4\Omega$ R = 12 Ω

At t = 0 $i_1 = \frac{5}{12}$

At $t = \infty$ $i_2 = \frac{5}{12} + \frac{5}{4} + \frac{5}{3} = \frac{40}{12}$

Required ratio $\frac{i_2}{i_1} = 8$

16. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

Sol. **(3)**

$$u = \frac{2\pi r^2 g\left(p - \sigma\right)}{9\pi\eta}$$

$$\frac{V_1}{V_2}=\frac{r_1^2\left(P-\sigma_1\right)\eta_2}{r_2^2\left(P-\sigma_2\right)\eta_1}$$

$$=\frac{\left(1\times10^{-2}\right)^2\times\left(8-0.8\right)\times2}{\left(0.5\times10^{-2}\right)\times\left(8-1.6\right)\times3}$$

$$=\frac{7.2\times2}{0.25\times6.4\times3}$$

= 3

17. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated *(P)* by the metal. The sensor has a scale that displays $\log_2$ *(P/P$_0$)*, where P_0 is a constant. When the metal surface is at a temperature of 487^0C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

Sol. **(9)**

$$P=\sigma e A\tau^4$$

$$\log_2\left(\frac{reA\left(487+273\right)^4}{P_0}\right)=1$$

$$\frac{\sigma e A}{P_0}=\frac{2}{\left(760\right)^4}$$

$$x=\log_2\left\{\frac{r\,e\,A\left(2767+273\right)}{p_0}\right\}=\log_2\left\{\frac{\left(3040\right)^4\times2}{\left(760\right)^4}\right\}$$

$$=\log_2\left(4\times2\right)=9$$

18. A hydrogen atom in its ground state is irradiated by light of wavelength $970\,\mathring{A}$. Taking $hc/e=1.237\times10^{-6}\,eV$ m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is

Sol. **(6)**

E = $\dfrac{12370}{970}$

= 12.752 ev

E_1 = -13.6 ev = $-13.6\dfrac{Z^2}{n^2}$

E_2 = - 3.4 ev

E_3 = - 1.51 ev

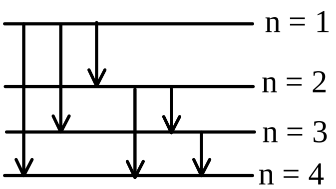
E_4 = - 0.85 ev

E_5 = - 0.544 ev

$E_4 - E_1 < E$

$E_5 - E_1 > E$

So, possible lines are



Total lines = 6

SECTION – II (CHEMISTRY)

Q. 19 The increasing order of atomic radii of the following Group 13 elements is

(A) $Al < Ga < In < Tl$ (B) $Ga < Al < In < Tl$

(C) $Al < In < Ga < Tl$ (D) $Al < Ga < Tl < In$

Sol: **(B)**

$Tl > In > Al > Ga$

'Ga' size is small due to poor shielding of d- orbital.

Q. 20 Among $\left[Ni(CO)_4\right], \left[NiCl_4\right]^{2-}, \left[Co(NH_3)_4 Cl_2\right]Cl, Na_3\left[CoF_6\right], Na_2O_2$ and CsO_2, the total number of paramagnetic compounds is

(A) 2 (B) 3 (C) 4 (D) 5

Sol: **(B)**

$\left[Ni(CO)_4\right] sp^3 \mu = 0$

$\left[NiCl_4\right]^{2-}$ sp^3 $\mu = \sqrt{8}$ B.M.

$\left[Co(NH_3)_4 Cl_2\right]Cl$ $\mu = 0$

Most of the Co^{+3} complexes are low spin

$\left[CoF_6\right]^{3-}$ $\mu = \sqrt{24}$ B.M.

Na_2O_2 diamagnetic

KO_2 paramagnetic one unpaired electron

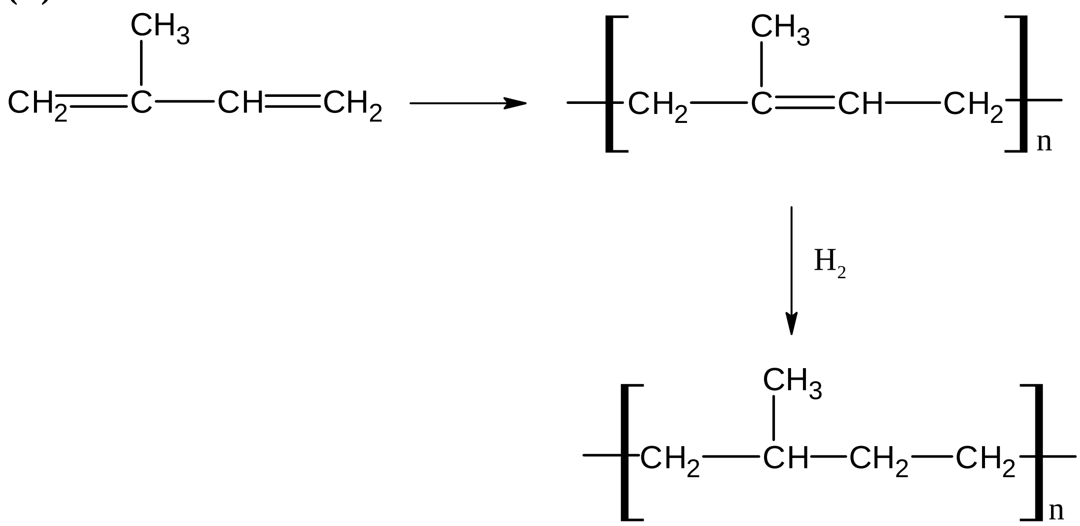
So total three paramagnetic substances.

Q. 21 On complete hydrogenation, natural rubber produces

(A) ethylene-propylene copolymer (B) vulcanized rubber

(C) polypropylene (D) polybutylene

Sol: **(A)**



Q. 22 One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings (ΔS_{surr}) in $J K^{-1}$ is

(1 L atm = 101.3 J)

(A) 5.763 (B) 1.013 (C) – 1.013 (D) – 5.763

Sol: **(C)**

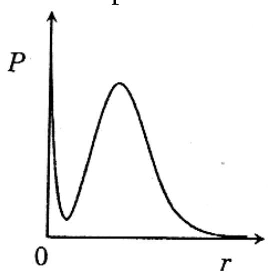
$Q_{sys} = +3(2-1) = 3\,L.atm$

$\Delta S_{surr} = \dfrac{-3\times 101.325}{300}$

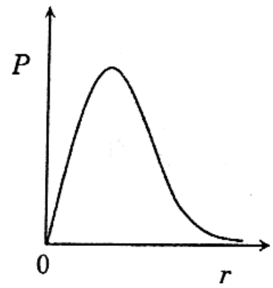
$= -1.01325$

Q. 23 P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of P on r is

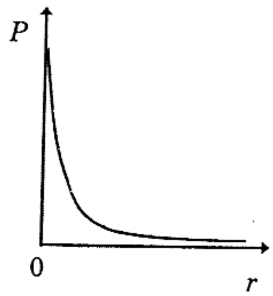
(A)



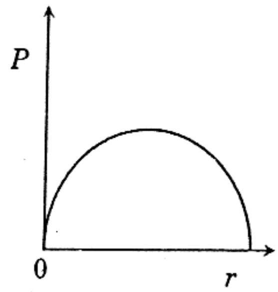
(B)



(C)



(D)



Sol: **(B)**

Q. 24 The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is (are)

(A) $CuCl_2$ (B) $BaCl_2$ (C) $Pb(OOCCH_3)_2$ (D) $Na_2[Fe(CN)_5NO]$

Sol: **(A)**

(A) $CuCl_2 + S^{2-} \longrightarrow Cus\downarrow$
Black ppt

$CusO_4$ is water soluble

(D) $Na_2[Fe(CN)_5NO] + Na_2S \longrightarrow Na_4[Fe(CN)_5NOS]$
Violet colour

Q. 25 A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, Z > 20. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is (are)

(A) β^- – decay(β emission) (B) orbital or K-electron capture

(C) neutron emission (D) β^+-decay (positron emission)

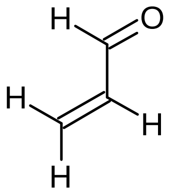
Sol: **(BD)**

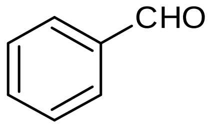
No. of neutrons have to increased

$p \rightarrow n + e^+ + \nu$

$p + e \rightarrow n + \nu$

Q. 26 Positive Tollen's test is observed for

(A)


(B)


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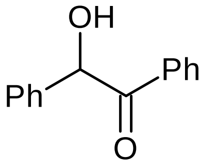

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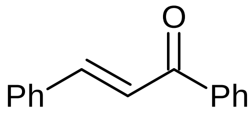


Sol: **(ABC)**

Give positive tollen's test

$$\underset{(\alpha\text{-Hydroxy ketone})}{\mathrm{Ph-\underset{\displaystyle |\atop OH}{CH^{\alpha}}-\underset{\displaystyle \|\atop O}{C}-Ph}} \longrightarrow \mathrm{Ph-\underset{\displaystyle \|\atop O}{C}-\underset{\displaystyle \|\atop O}{C}-Ph} + \mathrm{Ag}\downarrow \text{ mirror}$$

Q. 27 The correct statement(s) about the following reaction sequence is (are)

$$\text{Cumene}(C_9H_{12})\xrightarrow[\text{ii) } H_3O^+]{\text{i) } O_2} P \xrightarrow{CHCl_3/NaOH} Q(\text{major}) + R(\text{minor})\quad Q\xrightarrow[PhCH_2Br]{NaOH} S$$

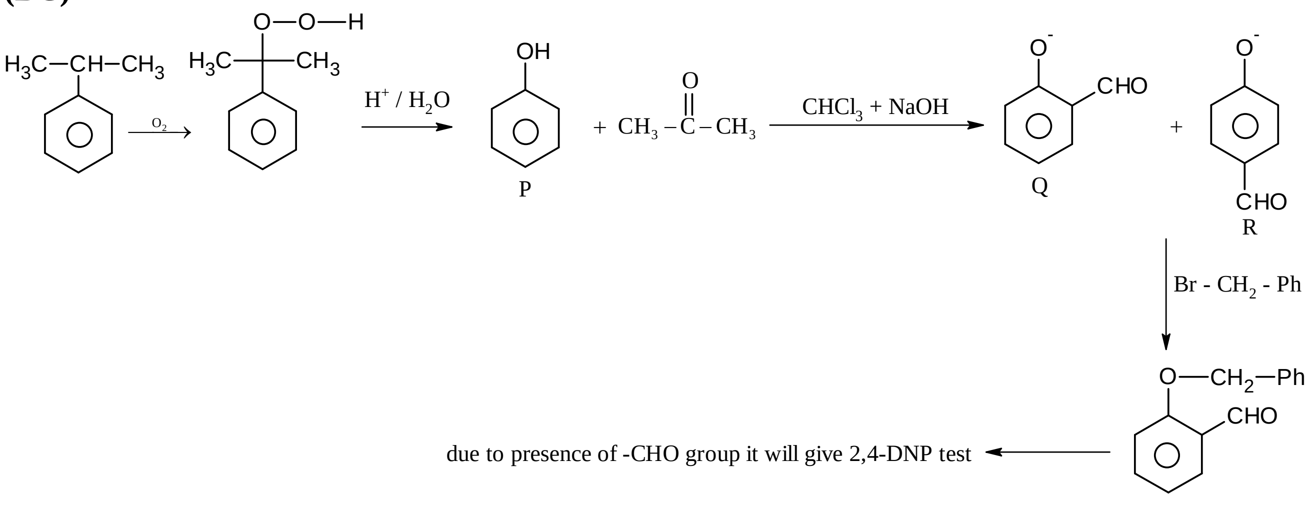
(A) R is steam volatile

(B) Q gives dark violet coloration with 1% aqueous $FeCl_3$ solution

(C) S gives yellow precipitate with 2, 4-dinitrophenylhydrazine

(D) S gives dark violet coloration with 1% aqueous $FeCl_3$ solution

Sol: **(BC)**



Q. 28 The crystalline form of borax has

(A) tetranuclear $\left[B_4O_5(OH)_4\right]^{2-}$ unit

(B) all boron atoms in the same plane

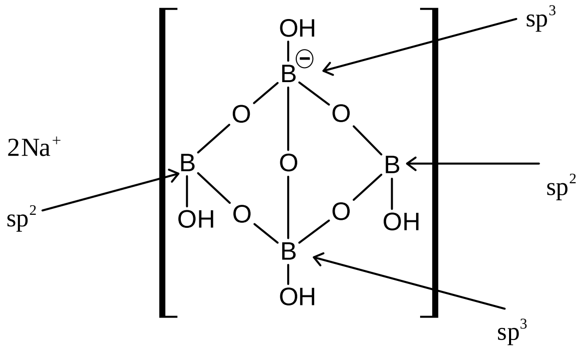
(C) equal number of sp^2 and sp^3 hybridized boron atoms

(D) one terminal hydroxide per boron atom

Sol: **(ACD)**

(A) Borax contains $Na_2\left[B_4(OH)O_5\right]$

(C)



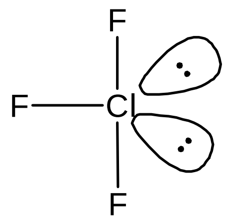
(D) each Boron contain one $O-H$

Q. 29 The compound(s) with twolone pairs of electrons on the central atom is(are)

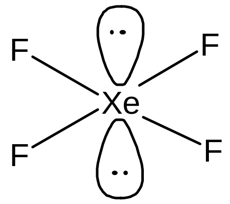
(A) BrF_5 (B) ClF_3 (C) XeF_4 (D) SF_4

Sol: **(BC)**

(B)



(C)



Q. 30 According to the Arrhenius equation,

(A) a high activation energy usually implies a fast reaction.

(B) rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy.

(C) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant.

(D) the pre-exponential factor is a measure of the rate at which collisions occurs irrespective of their energy.

Sol: **(BCD)**

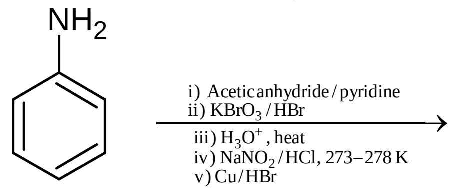
High Ea $\Rightarrow$ slow reaction

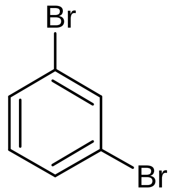
$$\log\frac{K_2}{K_1}=\frac{Ea}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

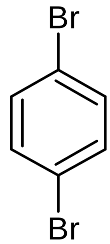
Higher Ea stronger temperature dependence

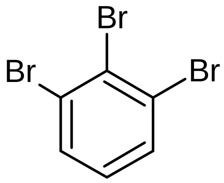
A depends on rate of collisions & temperature dependence is negligible.

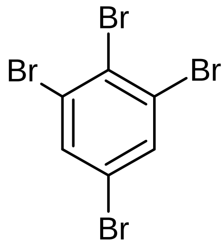
Q. 31 The product(s) of the following reaction sequence is (are)



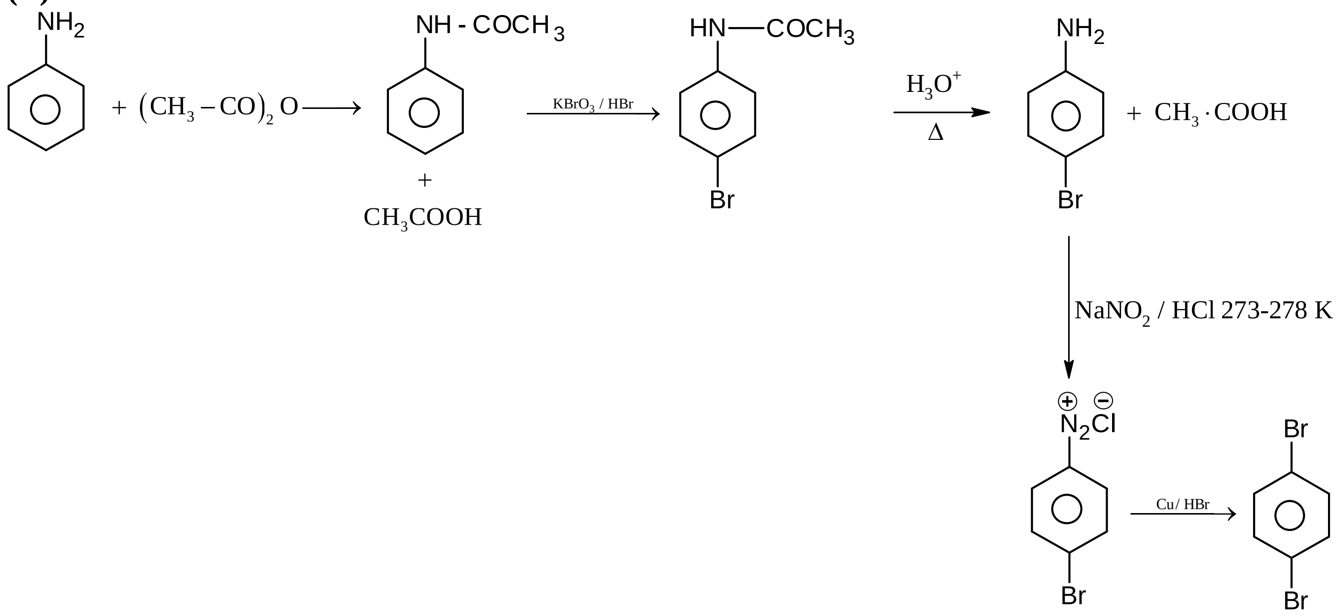
(A)


(B)


(C)


(D)


Sol: **(B)**



Q. 32 The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases *x* times. The value of *x* is

Sol: **(4)**

Mean free path $\lambda \propto \frac{T}{p}$

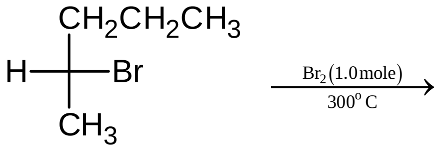
Mean speed $\propto \sqrt{T}$

$$D = \frac{1}{2}\lambda\, v_{avg}$$

$$D \propto \frac{T^{3/2}}{p}$$

So $\frac{4^{3/2}}{2} = 4$

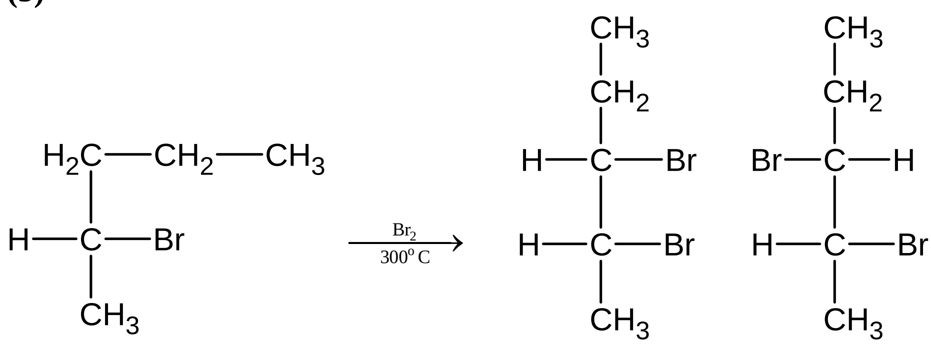
Q. 33 In the following monobromination reaction, the number of possible chiral products is



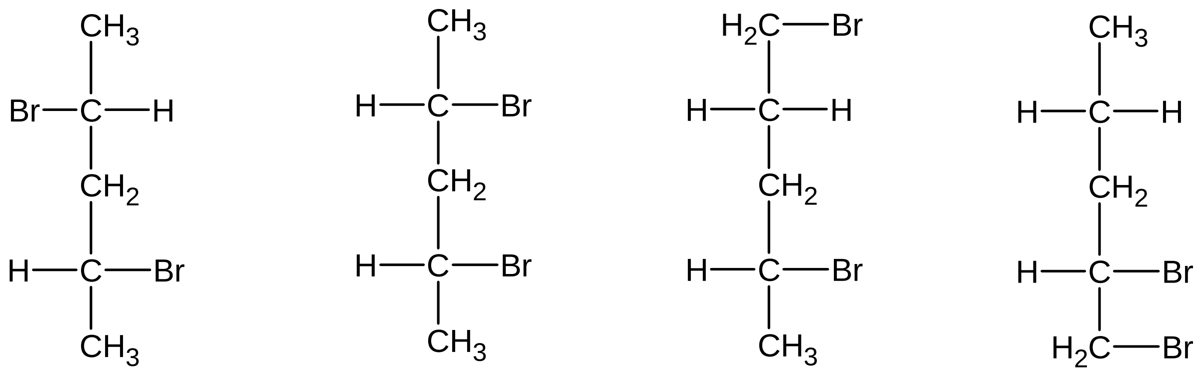
(1.0 mole)

(enantiomerically pure)

Sol: **(5)**



chiral chiral

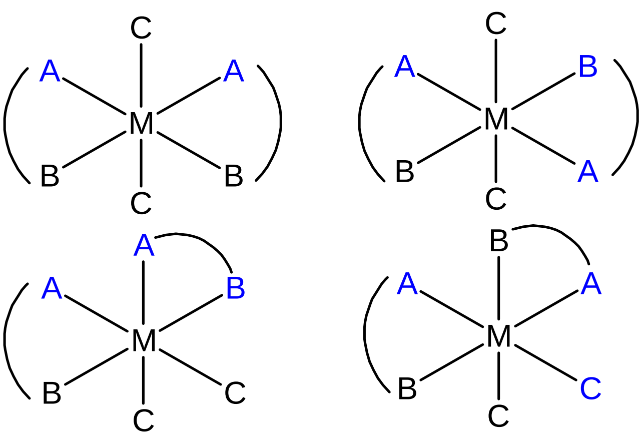


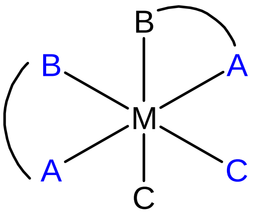
chiral Achiral chiral chiral

Q. 34 The number of geometric isomers possible for the complex $\left[CoL_2Cl_2\right]^{-}\left(L = H_2NCH_2CH_2O^{-}\right)$ is

Sol: **(5)**

$\left[M(AB)_2C_2\right]$ type





Q. 35 The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is

Sol: **(9)**

$$m(\text{molality}) = M(\text{molarity})$$

$$\frac{0.1}{\left(\frac{0.9M_{solvent}}{1000}\right)} = \frac{0.1}{\left(\frac{0.1M_{solute}+0.9M_{solvent}}{2\times1000}\right)}$$

$$\frac{1}{9M_{solvent}} = \frac{2}{M_{solute}+9M_{solvent}}$$

$$\frac{M_{solute}}{M_{solvent}} = 9$$

Q. 36 In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosuphate anions to produce **X** moles of a sulphur containing product. The magnitude of **X** is

Sol: **(6)**

$$MnO_4^- + S_2^{+2}O_3^{2-} \rightarrow S^{+6}O_4^{2-} + Mn^{4+}$$

n- factor 3 4

$$(n_E)_{MnO_4^-} = (n_E)_{SO_4^{2-}}$$

$$8\times3 = n_{SO_4^{2-}}\times4$$

$$n_{SO_4^{2-}} = 6$$

SECTION – III (MATHEMATICS)

Q 37. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P (computer turns out to be defective given that it is produced in plant T_1)

= 10 P (computer turns out to be defective given that it is produced in plant T_2),

Where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Sol: (C)

$P(\text{defective}/T_1) = 10P(\text{defective}/T_2)$

$P\left(\frac{T_2}{ND}\right) = \frac{P(T_2 \cap ND)}{P(ND)}$ (ND Means Non- Defective)

$= \frac{\frac{4}{5}\times(1-x)}{\frac{4}{5}(1-x)+\frac{1}{5}(1-10x)}$

$= \frac{4-4x}{5-14x}$

$P(\text{Defective}) = \frac{4}{5}\times x + \frac{1}{5}\times 1 - x = \frac{7}{100}$

$\Rightarrow x = \frac{1}{40}$

$\frac{4-\frac{1}{10}}{5-\frac{14}{40}} = \frac{39\times 2}{93} = \frac{78}{93}$

Q 38. The least value of $\alpha \in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all x > 0, is

(A) $\frac{1}{64}$ (B) $\frac{1}{32}$ (C) $\frac{1}{27}$ (D) $\frac{1}{25}$

Sol: (C)

$\frac{4ax^2 + \frac{1}{2x} + \frac{1}{2x}}{3} \geq (a)^{1/3}$

By A.M$-$G.M.

$\therefore$ For least value of a

$3a^{1/3} \geq 1 \Rightarrow a \geq \frac{1}{27}$

Q 39. Let $S=\left\{x\in(-\pi,\pi): x\neq 0,\pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x+\operatorname{cosec} x+2(\tan x-\cot x)=0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

Sol: (C)

$$-\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}+2\left(\frac{\sin^2 x-\cos^2 x}{\sin x\cos x}\right)=0$$

$$\frac{\sqrt{3}\sin x+\cos x}{2}=\cos(2x)$$

$$\cos\left(x-\frac{\pi}{3}\right)=\cos(2x)$$

$$x-\frac{\pi}{3}=2n\pi\pm 2x$$

(i) n positive

$$x=\frac{-6n\pi-\pi}{3}$$

$$x=-\frac{\pi}{3}$$

(ii) n negative

$$x=\frac{6n\pi+\pi}{9}$$

$$x=\frac{\pi}{9},\frac{7\pi}{9},\frac{-5\pi}{9}$$

$$\text{Sum}=-3\frac{\pi}{9}+\frac{\pi}{9}+\frac{7\pi}{9}-\frac{5\pi}{9}=0$$

Q 40. Let $-\frac{\pi}{6}<\theta<-\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2-2x\sec\theta+1=0$ and α_2 and β_2 are the roots of the equation $x^2+2x\tan\theta-1=0$. If $\alpha_1>\beta_1$ and $\alpha_2>\beta_2$, then $\alpha_1+\beta_2$ equals

(A) $2(\sec\theta-\tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Sol: (C)

$$2x\sec\theta=x^2+1$$

$$\therefore x=\frac{2\sec\theta\pm\sqrt{2^2\tan^2\theta}}{2}$$

$$\alpha_1,\beta_1=\sec\theta-\tan\theta$$

$$\sec\theta+\tan\theta$$

Now $x^2+2x\tan\theta-1=0$

$$\alpha_1+\beta_2=-2\tan\theta$$

Q 41. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320 (C) 260 (D) 95

Sol: (A)

If all girls are selected no. of ways ${}^6C_4\times 4$

[Captain can be selected is 4 ways]

$= 60$

If one boy is selected no. of ways $= {}^6C_3 \times {}^4C_1 \times 4 = 320$

Total ways $= 60 + 320 = 380$

Q 42. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

Sol: (A,C,D)

$2S = x + y + z$

$\frac{S-x}{4} = \frac{S-y}{3} = \frac{S-z}{2} = \lambda(\text{say})$

$S - x = 4\lambda$

$S - y = 3\lambda$

$S - z = 2\lambda$

$\Rightarrow 3S - (x + y + z) = 9\lambda$

$r = \frac{\Delta}{S} = \frac{\sqrt{S(S-a)(S-b)(S-c)}}{9\lambda}$

$r = \frac{6 \times \sqrt{6}\lambda}{9} = \frac{2\lambda\sqrt{6}}{3}$

Area $= \frac{8\pi}{3} = \pi\left(\frac{2\lambda\sqrt{6}}{3}\right)^2$

$\frac{8\pi}{3} = \pi \times \frac{4\lambda^2.6}{3}$

$\Rightarrow \lambda = 1$

Q 43. A solution curve of the differential equation $\left(x^2 + xy + 4x + 2y + 4\right)\frac{dy}{dx} - y^2 = 0, x > 0$, passes through the point (1, 3). Then the solution curve

(A) intersects y = x + 2 exactly at one point

(B) intersects y = x + 2 exactly at two points

(C) intersects $y = (x+2)^2$

(D) does NOT intersect $y = (x+3)^2$

Sol: (A,D)

$\left[(x+2)^2 + y(x+2)\right]\frac{dy}{dx} = x^2$

$$\left(x+2+y\right).\left(x+2\right)\frac{dy}{dx}=y^2$$
$$x+2=t$$
$$\left(t+y\right).t\frac{dy}{dt}=y^2$$
$$\frac{dy}{dt}=\frac{y^2}{t^2+ty}$$
$$y=v.t$$
$$\frac{dy}{dt}=v+t\frac{dv}{dt}$$
$$t\frac{dv}{dt}=\frac{v^2}{1+v}-v$$
$$t\frac{dv}{dt}=\frac{-v}{1+v}$$
$$\int\frac{\left(1+v\right)dv}{v}=\int\frac{dt}{t}$$
$$=\ln v+v=-\ln t+c$$
$$\Rightarrow\ln\left(\frac{y}{x+2}\right)+\left(\frac{y}{x+2}\right)=-\ln\left(x+2\right)+c$$
$$\Rightarrow 1=-\ln 3+c$$
$$\Rightarrow\ln\left(\frac{y}{3}\right)=\frac{-y}{\left(x+2\right)}+1$$

(C) $2\ln\frac{\left(x+2\right)}{\sqrt{3}}+\left(x+2\right)-1=0$

$2+3e-1$

$$\ln\left(\frac{x+2}{\sqrt{3}}\right)=-\frac{\left(x+1\right)}{2}$$
$$\frac{1}{x+2}+\frac{1}{2}>0$$

No solution

(D) $2\ln\left(\frac{x+3}{\sqrt{3}}\right)+\frac{\left(x+1\right)}{2}=0$

Q 44. Consider a pyramid OPQRS located in the first octant $\left(x\ge 0, y\ge 0, z\ge 0\right)$ with O is origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S in directly above the mid-point T of diagonal OQ such that TS = 3. Then

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is x – y = 0

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Sol: (B,C,D)

(i) $\cos\theta = \dfrac{\left(3\hat{i}+3\hat{j}\right).\left(\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+3\hat{k}\right)}{3\sqrt{2}\sqrt{\frac{27}{2}}}$

$$\frac{\frac{9}{2}+\frac{9}{2}}{3\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

(ii) $ax + by + cz = 0$

$3a + 4b = 0$

$\frac{9}{2} + \frac{b}{2} + 3c = 0$

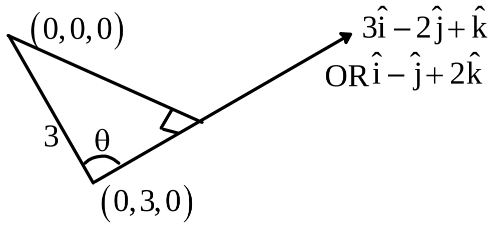
$x = y$

(iii) $(3,0,0), x - y = 0$

$\frac{3}{\sqrt{2}}$

(iv) Equation of line , $\left(\frac{3}{2},\frac{3}{2},3\right) \& (0,3,0)$

$$\frac{x}{\frac{3}{2}} = \frac{y-3}{-\frac{3}{2}} = \frac{z}{3}$$



$\cos\theta = \frac{1}{\sqrt{6}}$

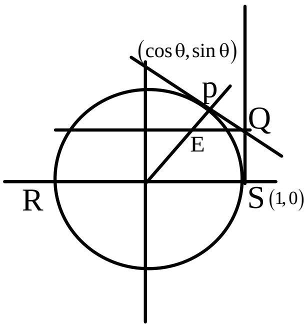
$\sin\theta = \frac{\sqrt{5}}{\sqrt{6}}$

$p = \frac{3\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{15}}{\sqrt{2}}$

Q 45. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4},\frac{1}{2}\right)$ (C) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4},-\frac{1}{2}\right)$

Sol: (A,C)



$x\cos\theta + y\sin\theta = 1$

$y = \dfrac{1-\cos\theta}{\sin\theta}$

$x = \dfrac{1-\cos\theta}{\sin\theta}.\dfrac{\cos\theta}{\sin\theta} = \dfrac{\cos\theta}{1+\cos\theta}$

$x = \dfrac{\cos\theta}{1+\cos\theta}, y = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

$\sec^2\theta = 1 + \dfrac{4^2}{x^2}$

$\cos^2\theta = \dfrac{x^2}{x^2+4^2}$

$x = \dfrac{\dfrac{x}{\sqrt{x^2+4^2}}}{1+\dfrac{x}{\sqrt{x^2+4^2}}} \Rightarrow \dfrac{1}{4}$

$x + \sqrt{x^2+4^2} = 1$

$\dfrac{1}{4} + \dfrac{1}{16} + \dfrac{1}{4}$

Q 46. Let $f:\mathbb{R}\to\mathbb{R}$, $g:\mathbb{R}\to\mathbb{R}$ and $h:\mathbb{R}\to\mathbb{R}$ be differentiable functions such that $f(x) = x^3+3x+2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x\in\mathbb{R}$, Then

(A) $g'(2) = \dfrac{1}{15}$ (B) $h'(1) = 666$ (C) $h(0) = 16$ (D) $h(g(3)) = 36$

Sol: (B,C)

(A) $g'(f(x)).f'(x) = 1$

$g'(2) = \dfrac{1}{f'(0)} = \dfrac{1}{3}$

(B) $h'(g(g(x)))g'(g(x)).g'(x) = 1$

$g(g(x)) = 1$

$g(x) = g^{-1}(1) = 6$

$x = g^{-1}(6) = 236$

(C) $h'(1).g'(g(236))g'(236) = 1$

$g'(236).f'(236) = 1$

$\left(\because g'\left(g\left(236\right)\right)=\frac{1}{6}\right)$

$g'\left(236\right)=\frac{1}{111}$

(D) $g\left(x\right)=3\Rightarrow x=g^{-1}\left(3\right)=f\left(3\right)=38$

$x=27+9+2=38$

$h\left(g\left(3\right)\right)=38$

Q 47. Let $f:\left(0,\infty\right)\to\mathbb{R}$ be a differentiable function such that $f'\left(x\right)=2-\frac{f\left(x\right)}{x}$ for all $x\in\left(0,\infty\right)$ and $f\left(1\right)\neq 1$. Then

(A) $\lim_{x\to 0+} f'\left(\frac{1}{x}\right)=1$

(B) $\lim_{x\to 0+} x f\left(\frac{1}{x}\right)=2$

(C) $\lim_{x\to 0+} x^2 f'\left(x\right)=0$

(D) $\left|f\left(x\right)\right|\le 2$ for all $x\in\left(0,2\right)$

Sol: (A)

$\int\left(x f'\left(x\right)+f\left(x\right)\right)dx=\int 2x\,dx$

$x f\left(x\right)=x^2+c$

$\Rightarrow f\left(x\right)=x+\frac{c}{x}$

$f'\left(x\right)=1-\frac{c}{x^2}$

$f\left(1\right)\neq 1\Rightarrow c\neq 0$

(A) $\lim_{x\to 0^+} f'\left(\frac{1}{x}\right)1-cx^2=1$

(B) $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)=x\left[\left(\frac{1}{x}\right)+cx\right]=1+cx^2=0$

(C) $\lim_{x\to 0^+} x^2 f'\left(x\right)=\lim_{x\to 0^+} x^2\left[1-\frac{c}{x^2}\right]=-c$

(D) $\left|f\left(x\right)\right|=\left|x+\frac{c}{x}\right|$

If $c=0\Rightarrow\left|f\left(x\right)\right|\le 2$

Q 48. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in\mathbb{R}$, Suppose $Q=\left[q_{ij}\right]$ is a matrix such that PQ = kI, where $k\in\mathbb{R}, k\neq 0$ and I is the identity matrix of order 3. If $q_{23}=-\frac{k}{8}$ and $\det\left(Q\right)=\frac{k^2}{2}$, then

(A) $\alpha=0, k=8$

(B) $4\alpha-k+8=0$

(C) $\det\left(P\ \mathrm{adj}\left(Q\right)\right)=2^9$

(D) $\det\left(Q\ \mathrm{adj}\left(P\right)\right)=2^{13}$

Sol: (B,C)

$$P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$$

$|Q|=\frac{K^2}{2}$

PQ=KI

$|P||Q|=K^3$

$(20+12\alpha)\frac{K^2}{2}=K^3 \Rightarrow K=10+6\alpha$

$Q=KP^{-1}$

$Q=K\frac{1}{|P|}(\text{adj}\,P)$

Now compare q_{23} of Q

$q_{23}=\frac{K}{(20+12\alpha)}(-3\alpha-4)=\frac{-K}{8}(\text{given})$

Simplify to get $\alpha=-1$

Hence $K=4$

(B) $4\alpha-K+8=0\Rightarrow 4-4+8=0$ correct

(C) $\det(P(\text{adj}\,Q))=|P||\text{adj}\,Q|$

$$=|P||Q|^2=(20+12\alpha).\left(\frac{K^2}{2}\right)^2$$

$$=(8)(8)^2=2^9 \text{ correct}$$

(D) $\det(Q\,\text{adj}\,P)=|Q||P|^2$

$$=\left(\frac{K^2}{2}\right)(20+12\alpha)^2=(8)(8)^2=2^9 \text{ wrong option}$$

Q 49. The circle $C_1: x^2+y^2=3$, with centre at O, intersects the parabola $x^2=2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3=12$

(B) $R_2R_3=4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Sol: (A,B,C)

Solve $x^2=2y \,\&\, x^2+y^2=3$

$\therefore P(\sqrt{2},1)$

Tangents at point P

$xx_1+yy_1=3$

$\therefore \sqrt{2}x+y-3=0$

$R_2 \& R_3$ have centres $(0, y_1)$

P=r

$\left|\frac{y_1-3}{\sqrt{3}}\right| = 2\sqrt{2}$

$\Rightarrow |y_1-3| = 6$

$\Rightarrow y_1 = 9, -3$

$\therefore Q_2 = (0,9) \& Q_3 = (0,-3)$

(A) $\therefore Q_2Q_3 = 12$

To find $R_2 \& R_3$

Let $R_2R_3\left(a, 3-\sqrt{2}a\right)$

$\therefore$ lies on $\sqrt{2}x + y - 3 = 0$

$m_1m_2 = -1$

$\Rightarrow \left(\frac{6+\sqrt{2}a}{a}\right)\times\left(\sqrt{2}\right) = 1$

$= 6\sqrt{2} + 2a = -a$

$3a = -6\sqrt{2}$

$a = -2\sqrt{2}$

$R_2\left(-2\sqrt{2}, 7\right)$

Similarly $R_3 = \left(2\sqrt{2}, -1\right)$

(B) $R_2R_3 = \sqrt{\left(4\sqrt{2}\right)^2 + 8^2}$

$= \sqrt{32+64} = \sqrt{96} = 4\sqrt{6}$

(C) Area of $\Delta OR_2R_3 = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -2\sqrt{2} & 7 & 1 \\ 2\sqrt{2} & -1 & 1 \end{vmatrix}$

$= \frac{1}{2}\times\left(2\sqrt{2} - 14\sqrt{2}\right)$

$= \frac{-12\sqrt{2}}{2} = 6\sqrt{2}$

$= \frac{1}{2}\times 12 \times \sqrt{2}$

Q 50. The total number of distinct $x \in [0,1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x-1$ is

Sol: (1)

Let $F(x) = \int_0^x \frac{t^2}{1+t^4}dt - 2x + 1$

$F(0) = 1$

$\Rightarrow F'(x) = \frac{x^2}{1+x^4} - 2 < 0$

$\therefore F(1) < 0$

Since $F(x)$ in decreasing and $F(0)F(1) < 0$

$\therefore F(x) = 0$ has exactly one solution in $[0,1]$

Q. 51. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + ... + (1+x)^{49} + (1+mx)^{50}$ is $(3n+1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is

Sol: (5)

Coefficient of x^2 in $(1+x)^2 + (1+x)^3 + (1+x)^4 + ... + (1+mx)^{50}$

${}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^{49}C_2 + {}^{50}C_2 m^2 = (3n+1)\,{}^{51}C_3$

${}^{50}C_3 + {}^{50}C_2 m^2 = 3n.{}^{51}C_3 + {}^{51}C_3$

${}^{50}C_2 m^2 = 3n.{}^{51}C_3 + {}^{50}C_2$

${}^{50}C_2 \left(m^2 - 1\right) = 3n.{}^{51}C_3$

$n = 1^2 - 1 = 0$

$m^2 - 1 = 51.n$

$m^2 = 51.n + 1$

Hence m is smallest for $n = 5$

Q 52. Let $z = \dfrac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is

Sol: (1)

$$P^2 = \begin{bmatrix} (-1)^r \omega^r & (\omega^2)^s \\ (\omega^2)^s & \omega^r \end{bmatrix} \begin{bmatrix} (-1)^r \omega^r & (\omega^2)^s \\ (\omega^2)^s & \omega^r \end{bmatrix}$$

$$= \begin{vmatrix} \omega^{2r} + \omega^s & \left(1+(-1)^r\right)\omega^{2s+r} \\ \left(1+(-1)^r\right)\omega^{2r+s} & \omega^{2r} + \omega^s \end{vmatrix} = -I$$

$\omega^{2r} + \omega^s = -1$

$1 + (-1)^r = 0$

$r =$ odd Integer

If $r = 1, s = 2$, if $r = 3$, No value of s

Q 53. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim\limits_{x \to 0} \dfrac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

Sol: (7)

$$\underset{x \to 0}{\text{Lt}} \frac{x^2 \sin \beta x}{(\alpha x - \sin x)} = 1$$

$$\underset{x\to 0}{\text{Lt}}\frac{x^2\beta x}{(\alpha x-\sin x)}\frac{\sin\beta x}{\beta x}\left[\because \underset{x\to 0}{\text{Lt}}\frac{\sin\beta x}{\beta x}=1\right]$$

$$\underset{x\to 0}{\text{Lt}}\frac{\beta x^2}{x\left(\alpha-\frac{\sin x}{x}\right)}=1$$

$$\left[\text{For }\frac{0}{0},\alpha=\underset{x\to 0}{\text{Lt}}\frac{\sin x}{x}=1\right]$$

$$\Rightarrow \underset{x\to 0}{\text{Lt}}\frac{\beta x^2}{\frac{x-\sin x}{x}}=1$$

$$\Rightarrow \underset{x\to 0}{\text{Lt}}\frac{\beta x^2}{x-\sin x}=1$$

$$\therefore \underset{x\to 0}{\text{Lt}}\frac{x-\sin x}{x^3}=\frac{1}{6}$$

$$\therefore 6\beta=1$$

$$\therefore \beta=\frac{1}{6}$$

$$\therefore 6(\alpha+\beta)=6\times\frac{7}{6}=7$$

Q 54. The total number of distinct $x\in\mathbb{R}$ for which $\begin{bmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{bmatrix}=10$ is

Sol: (2)

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix}=10$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ 2x & 4x^2 & 1 \\ 3x & 9x^2 & 1 \end{vmatrix}+6x^3\begin{vmatrix} 1 & x & x^2 \\ 1 & 2x & 4x^2 \\ 1 & 3x & 9x^2 \end{vmatrix}=10$$

$$\Rightarrow (1+6x^3)(x-2x)(2x-3x)(3x-x)=10$$

$$\Rightarrow 12x^6+2x^3-10=0$$

$$\Rightarrow 6x^6+x^3-5=0$$

$$\Rightarrow 6x^6+6x^3-5x^3-5=0$$

$$6x^3(x^3+1)-5(x^3+1)=0$$

$$(6x^3-5)(x^3+1)=0$$

$$x^3=\frac{5}{6},\ x^3=-1$$

$$x=\sqrt[3]{\frac{5}{6}}\ ,\ \ x^2=-1$$