JEE MAIN – 2017 QUESTION PAPER & SOLUTION

CHEMISTRY

1. The freezing point of benzene decreases by $0.45^{o}C$ when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associated to form a dimer in benzene, percentage association of acetic acid in benzene will be : (K_f for benzene = 5.12 K kg mol $^{-1}$)

(1) 94.6% (2) 64.6% (3) 80.4% (4) 74.6%

Sol. (1)

$$0.45 = 5.12 \times \frac{0.2/60}{20/1000} \times i$$

$$i = \frac{0.45 \times 20 \times 60}{5.12 \times 0.2 \times 1000} = 0.527$$

$$1 - \frac{\alpha}{2} = 0.527 \Rightarrow \alpha = (1 - 0.523) \times 2$$

$$\alpha = 0.946 = 94.6\%$$

2. On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is :

(1) $[Co(H_2O)_5Cl]Cl_2.H_2O$ (2) $[Co(H_2O)_4Cl_2]Cl_2.2H_2O$

(3) $[Co(H_2O)_3Cl_3].3H_2O$ (4) $[Co(H_2O)_6]Cl_3$

Sol. (1)

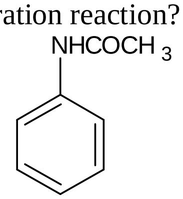
$$n_{ppt} = \frac{1.2 \times 10^{22}}{6 \times 10^{23}} = 2 \times 10^{-2}$$

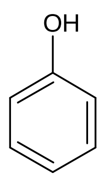
$$n_{compound} = 10 \times 10^{-3} = 10^{-2}$$

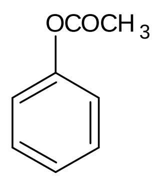
$\Rightarrow$ One mole compound gives two mores ppt.

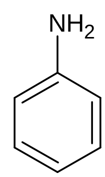
$$\Rightarrow \left[Co(H_2O)_5 Cl\right]Cl_2 \cdot H_2O$$

3. Which of the following compounds will form significant amount of *meta* product during mono-nitration reaction?

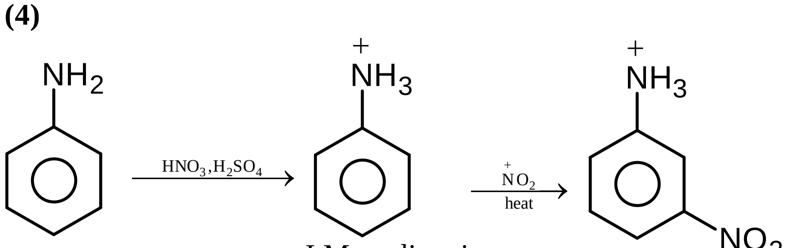
(1) 

(2) 

(3) 

(4) 

Sol. (4)



-I Meta directing group

(47%)

4. The product obtained when chlorine gas reactions with cold and dilute aqueous NaOH are :

(1) Cl^- and ClO_2^- (2) ClO^- and ClO_3^-

(3) ClO_2^- and ClO_3^- (4) Cl^- and ClO^-

Sol. (4)

$$\text{NaOH(Cold and Dilute)} + Cl_2 \downarrow \text{NaCl} + \text{NaClO}$$

5. Both lithium and magnesium display serval similar properties due to the diagonal relationship; however, the one which is incorrect, si :

(1) nitrates of both Li and Mg yield NO_2 and O_2 on heating
(2) both form basic carbonates
(3) both form soluble bicarbonates
(4) both form nitrides

Sol. (2) or (3)

'Mg'forms:-

$4MgCO_3.Mg(OH)_2.4H_2O$ which is basic carbonate

Lithium doesn't form basic carbonate.

6. A water sample has ppm level concentration of following anions

$F^- = 10; SO_4^{2-} = 100;\ NO_3^- = 50$

The anion / anions that makes / makes the water sample unsuitable for drinking is / are :

(1) Only SO_4^{2-} (2) only NO_3^-

(3) both SO_4^{2-} and NO_3^- (4) only F^-

Sol. (4)

Maximum permissible value for drinking:

ppm for $F^- = 1$ppm

ppm for $SO_4^{2-} = 5000$ppm

ppm ,, $NO_3^- > 50$ ppm

As given ppm for F^- is 10 ppm

That is why answer is (4)

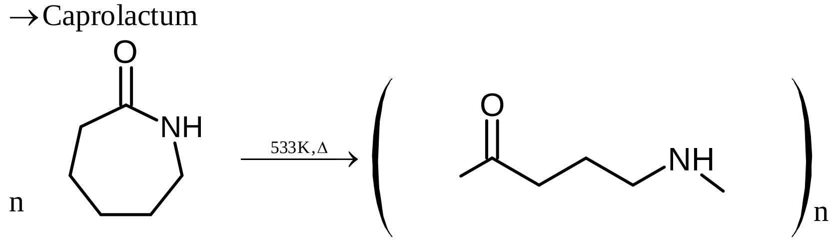
7. The formation of which of the following polymers involves hydrolysis reaction?

(1) Terylene (2) Nylon 6 (3) Bakelite (4) Nylon 6, 6

Sol. (2)

Nylon – 6 Formation involves hydrolysis

$\rightarrow$Caprolactum



Caprolactum Nylon - 6

Ans is (2)

8. The Tyndall effect is observed only when following conditions are satisfied:

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(b) The diameter of the dispersed particles is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude

(1) (b) and (c) (2) (a) and (d) (3) (b) and (d) (4) (a) and (c)

Sol. **(3)**

Condition for Tyndall effect are :

(1) The diameter of the dispersed particle is not much smaller than the wavelength of light used

(2) The refractive indices of the disperse phase and dispersion medium differ greatly in magnitude

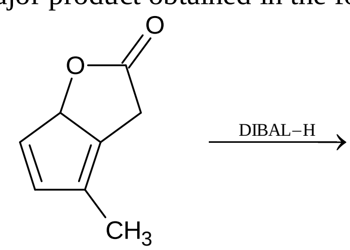
9. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is :

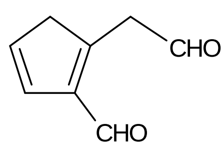
(1) 1.0 (2) 7.2 (3) 6.9 (4) 7.0

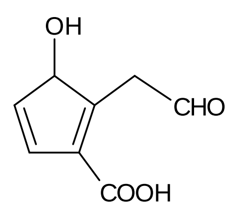
Sol. **(3)**

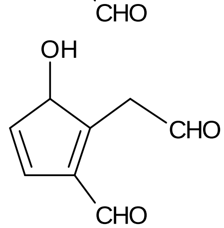
$$pH = 7 + \frac{1}{2} \times 3.2 - \frac{1}{2} \times 3.4$$
$$= 6.9$$

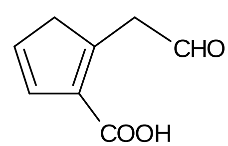
10. The major product obtained in the following reaction is :



(1) 

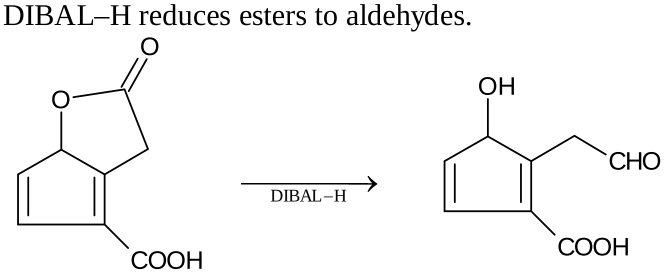
(2) 

(3) 

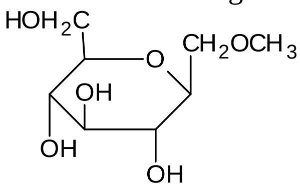
(4) 

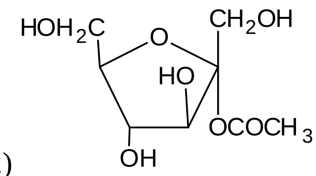
Sol. **(2)**

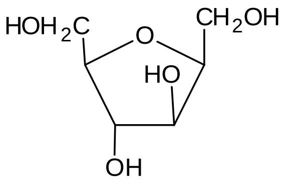
DIBAL–H reduces esters to aldehydes.

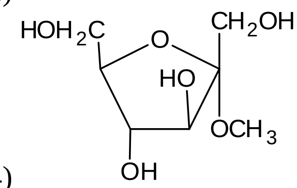


11. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

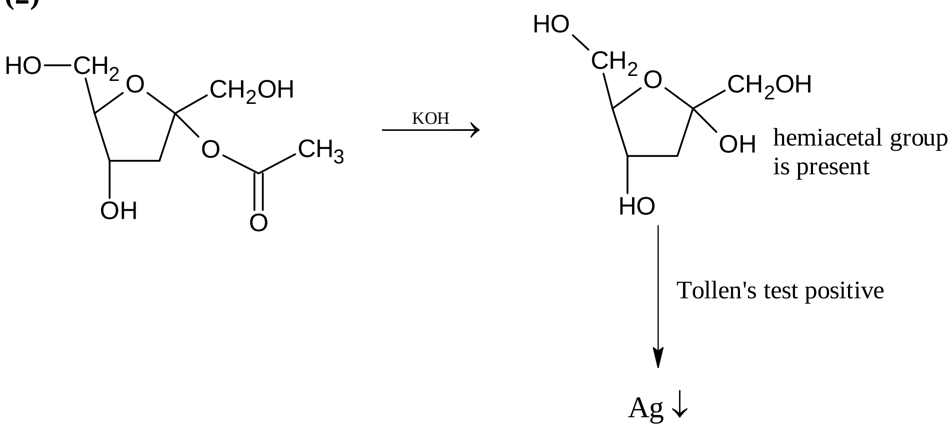
(1) 

(2) 

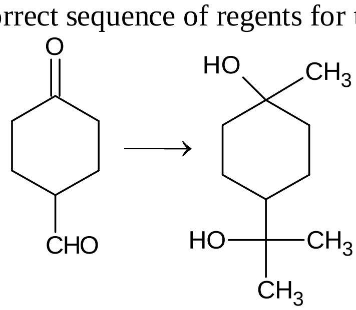
(3) 

(4) 

Sol. **(2)**



12. The correct sequence of regents for the following conversion will be :



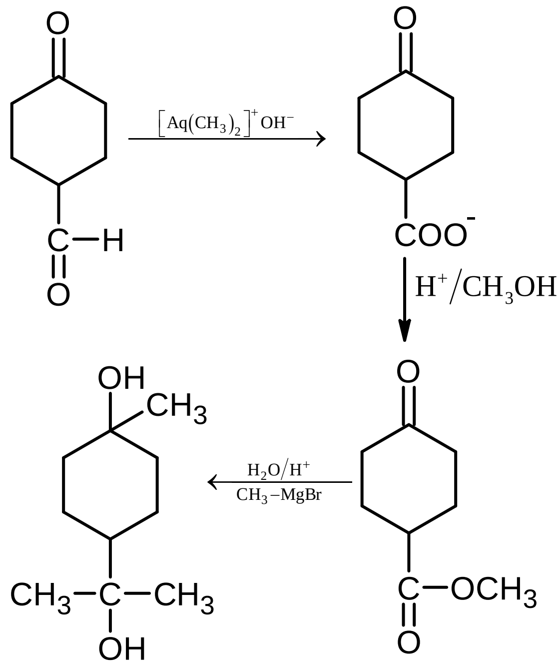
(1) $\left[Ag\left(NH_3\right)_2\right]^+ OH^-,\ CH_3MgBr, H^+/CH_3OH$

(2) $\left[Ag\left(NH_3\right)_2\right]^+ OH^-, H^+/CH_3OH,\ CH_3MgBr$

(3) $CH_3NgBr, H^+/CH_3\,OH, \left[Ag\left(NH_3\right)_2\right]^+ OH^-$

(4) $CH_3MgBr, \left[Ag\left(NH_3\right)_2\right]^+ OH^-, H^+/CH_3\,OH$

Sol. **(2)**



13. Which of the following species is not paramagnetic?

(1) B_2 (2) NO (3) CO (4) O_2

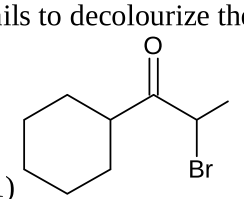
Sol. **(3)**

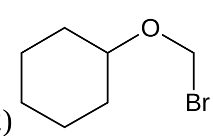
For CO

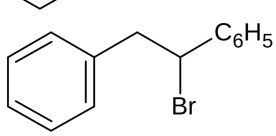
$$\sigma 1s^2, \sigma 1s^2, \sigma 2s^2, \begin{cases} \pi_{2Px}{}^2, \\ \pi_{2Py}{}^2 \end{cases} \sigma p_z^2, \sigma 2s^2$$

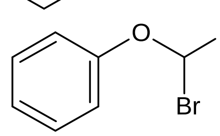
$\rightarrow$All electrons are paired.

14. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

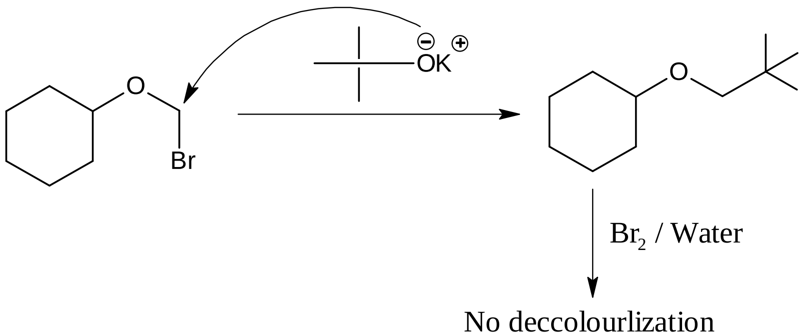
(1) 

(2) 

(3) 

(4) 

Sol. **(2)**



15. Which of the following reactions is an example of a redox reaction?

(1) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

(2) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

(3) $XeF_2 + O_2F_2 \rightarrow XeF_6 + O_2$

(4) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

Sol. **(2)**

oxidation

$$\overset{+4}{XeF_4} + \overset{+1}{O_2}F_2 \longrightarrow \overset{+6}{XeF_6} + \overset{0}{O_2}$$

Reduction

16. ΔU is equal to:

(1) Isothermal work

(2) Isochoric work

(3) Isobaric work

(4) Adiabatic work

Sol. **(4)**

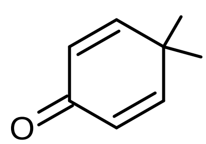
By 1st law of thermodynamics

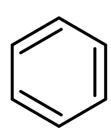
$\Delta U = q + \omega$

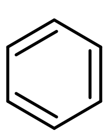
For adiabatic $q = 0$

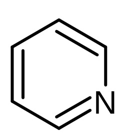
Hence $\Delta U = \omega$

17. Which of the following molecules is least resonance stabilized?

(1) 

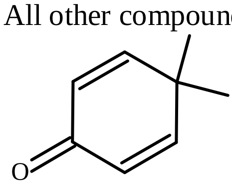
(2) 

(3) 

(4) 

Sol. **(1)**

All other compounds are aromatic except



18. The increasing order of the reactivity of the following halides for the S_N1 reaction is:

$$\underset{\substack{|\\ C\ell}}{CH_3CH}CH_2CH_3 \qquad\qquad CH_3CH_2CH_2Cl$$

(I) (II)

$p-H_3CO-C_6H_4-CH_2Cl$

(III)

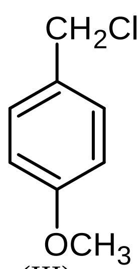
(1) $(II)<(III)<(I)$ (2) $(III)<(II)<(I)$

(3) $(II)<(I)<(III)$ (4) $(I)<(III)<(II)$

Sol. **(3)**

$$CH_3-\underset{\substack{|\\ C\ell}}{CH}-CH_3-CH_3 \qquad CH_3-CH_2-CH_2-Cl$$

(I) (II)



(III)

Carbocation would be most stable in (III) then (I) & (II).

$\therefore$ Rate of SN^1 reactivity is

$II<I<III$

19. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is

(1) 11.86 (2) 1186 (3) 84.3 (4) 118.6

Sol. **(3)**

$M_2CO_3+2HCl\rightarrow MCl+H_2O+CO_2$

$$\frac{n_{N_2CO_3}}{n_{CO_2}}=1$$

$$\frac{\frac{1}{2x+60}}{0.01186}=1$$

$$2x+60=\frac{1}{0.01186}$$

$x=12.15$

Molar mass of $N_2CO_3=2\times12.15+60$

20. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$ 'X' is:

(1) $Na_2C_2O_4$ (2) C_6H_5COONa (3) $HCOONa$ (4) CH_3COONa

Sol. **(1)**

$$Na_2C_2O_4 \xrightarrow{\text{conc.}H_2SO_4} CO + CO_2, C_2O_4^{2-} + Ca^{2+} \rightarrow \underset{\text{white}}{CaC_2O_4} \downarrow$$

$$C_2O_4^{2-} + MnO_4^{-} + H^{+} \rightarrow CO_2 \uparrow + Mn^{2+} + H_2O$$

21. The most abundant elements by mass in the body of a healthy human adult are:
Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ${}^{1}H$ atoms are replaced by ${}^{2}H$ atoms is:

(1) 10 kg (2) 15 kg (3) 37.5 kg (4) 7.5 kg

Sol. **(4)**

Weight of person $= 75\,Kg$

Hydrogen $= \frac{10}{100} \times 75 = 7.5\,Kg$

Atoms if ${}_{1}^{1}H = \frac{7.5 \times 10^{3}}{1} \times 6.022 \times 10^{23}$

New mass of ${}_{1}^{2}H =$ mass of 1 atom $\times$ No. of atoms

$$= \frac{0 \times 7.5 \times 10^{3} \times 6.022 \times 10^{23} \times 2}{6.023 \times 10^{23}}$$

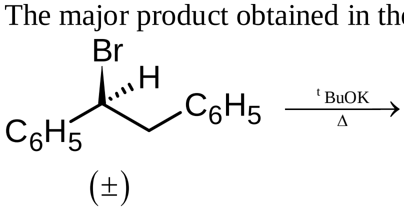
$$= 15 \times 10^{3}\,gm$$

Weight gain due to ${}^{2}H = 15 \times 10^{3} - 7.5 \times 10^{3}$

$$= 7.5 \times 10^{3}\,gm$$

$$= 7.5Kg$$

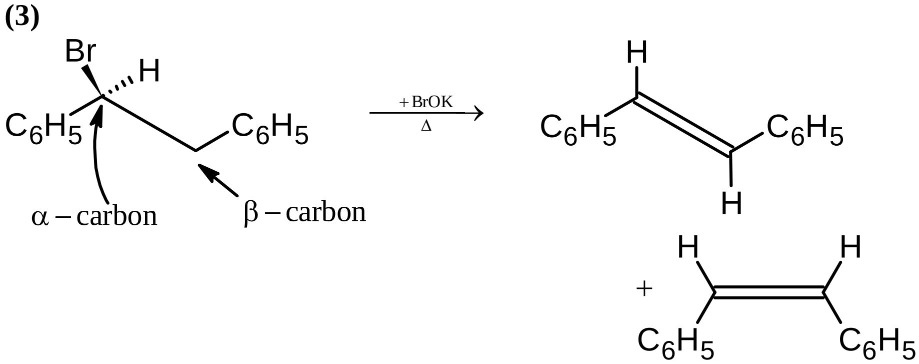
22. The major product obtained in the following reaction is:



(1) $(-)C_6H_5CH(O^tBu)CH_2C_6H_5$ (2) $(\pm)C_6H_5CH(O^tBu)CH_2C_6H_5$

(3) $C_6H_5CH=CHC_6H_5$ (4) $(+)C_6H_5CH(O^tBu)CH_2C_6H_5$

Sol. **(3)**



This is a typical example of E – 2 elimination

$\therefore$ Ans is (3)

23. Given

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)$;

$\Delta_r H^0 = -393.\text{kJ mol}^{-1}$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$;

$\Delta_r H^0 = -285.8\text{ kJ mol}^{-1}$

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g)$;

$\Delta_r H^0 = +890.3\text{ kJ mol}^{-1}$

Based on the above thermochemical equations, the value of $\Delta_r H^0$ at 298 K for the reaction $C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$ will be:

(1) $-144.0\text{ kJ mol}^{-1}$ (2) $+74.8\text{ kJ mol}^{-1}$

(3) $+144.0\text{ kJ mol}^{-1}$ (4) -74.8 kJ mol^{-1}

Sol. **(4)**

$C_{(graphite)} + O_2(g) \longrightarrow CO_2(g)$ .... (1)

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell)$ .... (2)

$CO_2(g) + 2H_2O(\ell) \longrightarrow CH_4(g) + 2O_2(g)$ .... (3)

$C_{(graphite)} + 2H_2(g) \longrightarrow CH_4(g)$ .... (4)

$1 + 2\times(2) + (3)$ gives (4)

$C_{(graphite)} + O_2(g) + 2H_2(g) + O_2(g) \longrightarrow CO_2(g) + 2H_2O(\ell) + CH_4(g)$

$+CO_2(g) + 2H_2O(\ell)$

$\Delta_r H = -393.5 - (285.8)\times 2 + 890.3$

$= -74.8\text{ KJ/mol}$

24. In the following reactions, ZnO is respectively acting as a / an:

(a) $ZnO + Na_2O \rightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \rightarrow ZnCO_3$

(1) acid and base (2) base and acid (3) base and base (4) acid and acid

Sol. **(1)**

$\underset{\text{Acidic}}{ZnO} + \underset{\text{Basic}}{Na_2O} \rightarrow Na_2ZNO_2$ $\underset{\text{Basic}}{ZNO} + \underset{\text{Acidic}}{CO_2} \rightarrow ZnCO_3$

25. The radius of the second Bohr orbit for hydrogen atom is:

(Planck's const. $h = 6.6262\times10^{-34}$ Js mass of electron $= 9.1091\times10^{-31}$ kg; charge of electron $e = 1.60210\times10^{-19}$ C; permittivity of vacuum $\epsilon_0 = 8.854185\times10^{-12}\text{ kg}^{-1}\text{m}^{-3}\text{A}^2$)

(1) $2.12\overset{0}{A}$ (2) $1.65\overset{0}{A}$ (3) $4.76\overset{0}{A}$ (4) $0.529\overset{0}{A}$

Sol. **(1)**

$$r_2 = \frac{4}{1} \times 0.529\,\mathring{A}$$

$$= 2.12\,\mathring{A}$$

26. Two reactions R_1 and R_2 have identical pre – exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to: $(R = 8.314\,J\,mol^{-1}K^{-1})$

(1) 4 (2) 8 (3) 12 (4) 6

Sol. **(1)**

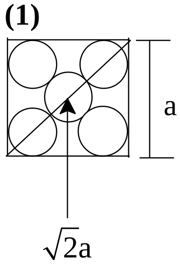
$$\frac{R_2}{R_1} = e^{\frac{10000}{8.314 \times 300}} = e^4$$

$$\ln\frac{R_2}{R_1} = 4$$

27. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be:

(1) $\frac{a}{\sqrt{2}}$ (2) 2a (3) $2\sqrt{2}a$ (4) $\sqrt{2}\,a$

Sol. **(1)**



$\Rightarrow$ The closest distance between two atoms in metallic crystal is $\frac{\sqrt{2}a}{2}$

$$= \frac{a}{\sqrt{2}}$$

$\therefore$ Ans. is (1)

28. The group having isoelectronic species is:

(1) O^-, F^-, Na^+, Mg^{2+} (2) $O^{2-}, F^-, Na^+, Mg^{2+}$

(3) O^-, F^-, Na, Mg^+ (4) O^{2-}, F^-, Na, Mg^{2+}

Sol. **(2)**

10 electrons are present in each species

29. Given

$E^0_{Cl_2/Cl^-} = 1.36\,V, E^0_{Cr^{3+}/Cr} - 0.74\,V$

$E^0_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\,V, E^0_{MnO_4^-/Mn^{2+}=1.51\,V}$.

Among the following, the strongest reducing agent is:

(1) Cl^- (2) Cr (3) Mn^{2+} (4) Cr^{3+}

Sol. **(2)**

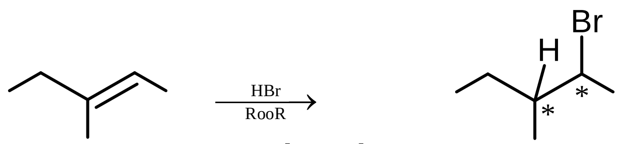
$E^{o}_{Cr/Cr^{3+}} = 0.74$

Oxidation potential is maximum hence stronger reducing agent.

30. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is:

(1) Four (2) Six (3) Zero (4) Two

Sol. **(1)**



Antimarkovinikov's

$\because$ In products two chiral carbon are generated & molecule is unsymmetric

$\therefore$ Total isomers are $2^2 = 4$

$\therefore$ Ans is (1)

MATHEMATICS

31. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equals

(1) 4 (2) -1 (3) -2 (4) 2

Sol. (4)

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x(\pi - x)}$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$$

Add

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2dx}{1-\cos^2 x}$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos ec^2 x dx$$

$$I = 2$$

32. Let $I_n = \int \tan^n x\, dx, (n > 1)$ .If $I_4 + I_6 = a\tan^5 c + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to

(1) $\left(\frac{1}{5}, -1\right)$ (2) $\left(-\frac{1}{5}, 0\right)$ (3) $\left(-\frac{1}{5}, 1\right)$ (4) $\left(\frac{1}{5}, 0\right)$

Sol. (4)

$$I_n = \int \tan^4 x\, dx (n > 1)$$

$$I_4 + I_6 = (\tan^4 x + \tan^6 x) dx$$

$$= \int \tan^4 x (\sec^2 x) dx$$

$$\tan x = 1, \sec^2 x = dx$$

$$= \int t^4 dt$$

$$= \frac{t^5}{5} + c$$

$$\frac{1}{5}\tan^4 x + C$$

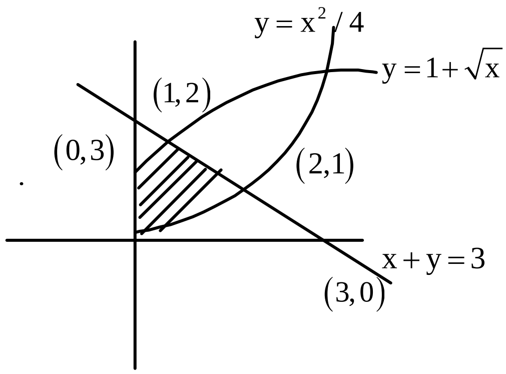
$$a = \frac{1}{5}, b = 0$$

33. The area (in sq.units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is:

(1) $\frac{7}{3}$ (2) $\frac{5}{2}$ (3) $\frac{59}{12}$ (4) $\frac{3}{2}$

Sol. **(2)**

$x \geq 0, x + y \leq 3, x^2 \leq 4y \ \& \ y \leq 1 + \sqrt{x}$



$$A = \int_0^1 \left(\left(1 + \sqrt{x}\right) - \frac{x^2}{4} \right) dx + \int_1^2 \left((3 - x) - \frac{x^2}{4} \right) dx$$

$$= \left(x + \frac{2}{3} x^{3/2} - \frac{x^3}{12} \right)_0^1 + \left(3x - \frac{x^2}{2} - \frac{x^3}{12} \right)_1^2$$

$$= \left(1 + \frac{2}{3} - \frac{1}{12} \right) + \left(6 - 2 - \frac{8}{12} - 2 + \frac{1}{2} + \frac{1}{12} \right)$$

$$= 2 + \frac{2}{3} - \frac{8}{12} + \frac{6}{12}$$

$$= 2 + \frac{2}{3} - \frac{2}{12}$$

$$= \frac{5}{2}$$

34. A bod contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:

(1) 4 (2) $\frac{6}{25}$ (3) $\frac{12}{5}$ (4) 6

Sol. **(3)**

$P(G) = \frac{3}{5}$

$P(y) = \frac{2}{5}$

Variance $= n\,pq$

Variance $= 10 \times \frac{3}{5} \times \frac{2}{5}$

$= \frac{12}{5}$

35. If $\left(2+\sin x\right)\frac{dy}{dx}+\left(y+1\right)\cos x=0$ and $y\left(0\right)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to

(1) $-\frac{1}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $-\frac{2}{3}$

Sol. (3)

$\left(2+\sin x\right)\frac{dy}{dx}+\left(y+1\right)\cos x=0$

$\int\frac{dy}{y+1}=-\int\frac{\cos x\,dx}{1+\sin x}$

$\ell n\left|y+1\right|=-\ell n\left|2+\sin x\right|+C$

Now, $y\left(0\right)=1\Rightarrow C=2\ell n2$

$\Rightarrow \ell n\left|y+1\right|=-\ell n\left|2+\sin x\right|+2\ell n\,2$

$\Rightarrow y+1=\frac{4}{2+\sin x}$

Now, $y\left(\frac{\pi}{2}\right)=\frac{4}{3}-1=\frac{1}{3}$

36. Let ω be a complex number such that $2\omega+1=z$ where $z=\sqrt{-3}$. If

$$\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}=3k$$

Then k is equal to:

(1) -1 (2) 1 (3) $-z$ (4) z

Sol. (3)

On solving $\omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$

$$\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}=3\omega\left(\omega-1\right)$$

$\Rightarrow k=\omega^2-\omega$

$=-1-\omega-\omega$

$=-z$

37. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. Let $\vec{c}$ be a vector such that $\left|\vec{c}-\vec{a}\right|=3$, $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|=3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to

(1) 5 (2) $\frac{1}{8}$ (3) $\frac{25}{8}$ (4) 2

Sol. (4)

$\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix}=2\hat{i}-2\hat{j}+\hat{k}$

Now, $\left|\vec{c}-\vec{a}\right|=3\Rightarrow \vec{c}^{\,2}-2\vec{c}.\vec{a}=0$, Now, $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|=3$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|\left|\vec{c}\right|\sin\theta = 3, \quad \Rightarrow 3\times\left|\vec{c}\right|\sin\left(\frac{\pi}{6}\right) = 3 \Rightarrow \left|\vec{c}\right| = 2$

$\Rightarrow \left|\vec{c}\right|^2 = 2\vec{c}.\vec{a} = 0 \Rightarrow \vec{c}.\vec{a} = 1$

38. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines $y = |x|$ is:

(1) $4\left(\sqrt{2}-1\right)$ (2) $4\left(\sqrt{2}+1\right)$ (3) $2\left(\sqrt{2}+1\right)$ (4) $2\left(\sqrt{2}-1\right)$

Sol. **(1)**

Let radius $= r$

Centre $(0, 4-r)$

$y - x = 0$

$\frac{4-r}{\sqrt{2}} = r$

$4 - r = \sqrt{2}r$

$\Rightarrow r = \frac{4}{\sqrt{2}+1} = 4\left(\sqrt{2}-1\right)$

39. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$,then $g(x)$ equals:

(1) $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$ (3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

Sol. **(3)**

$\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right) = \tan^{-1}\left(\frac{2\left(3x\sqrt{x}\right)}{1-\left(3x\sqrt{x}\right)^2}\right)$

Put $3x\sqrt{x} = \tan\theta$

Then $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right) = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \tan^{-1}\left(\tan 2\theta\right)$

$\Rightarrow 2\theta = 2\tan^{-1} 3x\sqrt{x}$

$\Rightarrow y = 2\tan^{-1} 3x\sqrt{x}$

Differentiating

$\frac{dy}{dx} = \frac{9\sqrt{x}}{1+9x^3}$

$\therefore g(x) = \frac{9}{1+9x^3}$

40. If two different numbers are taken from the set $\{0,1,2,3,.....,10\}$; then the probability that their sum as well as absolute difference are both multiple of 4 , is:

(1) $\frac{14}{45}$ (2) $\frac{7}{55}$ (3) $\frac{6}{55}$ (4) $\frac{12}{55}$

Sol. **(3)**

Case I Both number are of form $4K = {}^3C_2$

Case II Both number are of form $4k+2 = {}^3C_2$

Probability $= \frac{6}{55}$

41. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals:

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$ (3) $\frac{1}{24}$ (4) $\frac{1}{16}$

Sol. **(4)**

$$\ell t_{x\to\frac{\pi}{2}} \frac{(\cos x - \cos x \sin x)}{\left(\frac{\pi}{2} - x\right)^3 (\sin x)}$$

$$\frac{1}{8} \ell t_{x\to\frac{\pi}{2}} \frac{\cos x}{\left(\frac{\pi}{2} - 4\right)^3}[1 - \sin x]$$

$$\frac{1}{8} \ell t_{x\to\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} \frac{1 - \cos\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)^2}$$

$$\frac{1}{8} \times 1 \times \frac{1}{2} \qquad = \frac{1}{16}$$

42. The value of

$\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + ... + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$ is:

(1) $2^{20} - 2^9$ (2) $2^{20} - 2^{10}$ (3) $2^{21} - 2^{11}$ (4) $2^{21} - 2^{10}$

Sol. **(2)**

$$\left({}^{21}C_1 + {}^{21}C_2 + ... {}^{21}C_{10}\right)$$

$$-\left({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + ... {}^{10}C_{10}\right)$$

$$= \left(\frac{2^{21}}{2} - 1\right) - \left(2^{10} - 1\right)$$

$$= 2^{20} - 2^{10}$$

43. For three events A,B and C, P (Exactly one of A or B occurs)
=P (Exactly one of B or C occurs)
=P (Exactly one of C or A occurs) $\frac{1}{4}$ and P (All the three events occur simultaneously) $\frac{1}{16}$.
Then the probability that at least one of the events occurs, is:

(1) $\frac{7}{64}$ (2) $\frac{3}{16}$ (3) $\frac{7}{32}$ (4) $\frac{7}{16}$

Sol. **(4)**

$$P(A)+P(B)-2P(A\cap B)=\frac{1}{4} \qquad \_\_\_\_\_\_\_(1)$$

$$P(B)+P(C)-2P(B\cap C)=\frac{1}{4} \qquad \_\_\_\_\_\_\_\_(2)$$

$$P(A)+P(C)-2P(A\cap C)=\frac{1}{4} \qquad \_\_\_\_\_\_\_\_\_(3)$$

$$P(A\cap B\cap C)=\frac{1}{16}$$

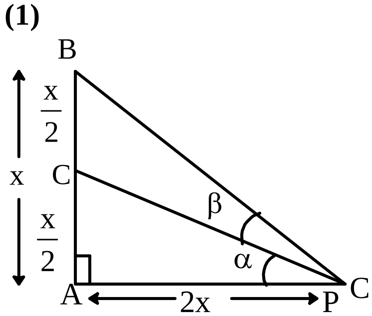
Adding (1) +(2)+ (3) $\Rightarrow P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)=\frac{3}{8}$

$$P(A\cup B\cup C)=\frac{3}{8}+\frac{1}{16}=\frac{7}{16}$$

44. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP=2AB$. If $\angle BPC=\beta$, then $\tan\beta$ is equal to:

(1) $\frac{2}{9}$ (2) $\frac{4}{9}$ (3) $\frac{6}{7}$ (4) $\frac{1}{4}$

Sol. **(1)**



$$\tan\alpha=\frac{x/2}{2x}=\frac{1}{4}$$

$$\tan(\alpha+\beta)=\frac{1}{2}=\frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta}$$

$$\frac{1}{2}-\frac{1}{8}\tan\beta=\frac{1}{4}+\tan\beta$$

$$\tan\beta=\frac{2}{9}$$

45. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x=-4$, then the equation of the normal to its at $\left(1,\frac{3}{2}\right)$ is:

(1) $4x+2y=7$ (2) $x+2y=4$ (3) $2y-x=2$ (4) $4x-2y=1$

Sol. **(4)**

$e=\frac{1}{2}$

$\frac{a}{e}=4 \Rightarrow a=2$

$b=\sqrt{a^2\left(1-e^2\right)}=\sqrt{4\left(1-\frac{1}{4}\right)}=\sqrt{3}$

$\left(a\cos\theta, b\sin\theta\right)=\left(1,\frac{3}{2}\right) \Rightarrow \theta=\frac{\pi}{3}$

Normal: $ax\sec\theta - by\operatorname{cosec}\theta = a^2-b^2$

$\Rightarrow 4x-2y=1$

46. If, for a positive integer n, the quadratic equation,

$x\left(x+1\right)+\left(x+1\right)\left(x+2\right)+.....+\left(x+\overline{n-1}\right)\left(x+n\right)=10n$

Has two conservative integral solutions, then n is equal to:

(1) 10 (2) 11 (3) 12 (4) 9

Sol. **(2)**

$nx^2+n^2x+\frac{n\left(n^2-1\right)}{3}=10n$

$x^2+nx+\frac{\left(n^2-1\right)}{3}-10=0$

$\left|\alpha-\beta\right|=1$

$n^2-4\left(\frac{n^2-31}{3}\right)=1$

$3n^2-4n^2+124=3$

$\Rightarrow n=11$

47. The following statement

$\left(p\to q\right)\to\left[\sim\left(\sim P\to q\right)\to q\right]$ is:

(1) equivalent to $p\to\sim q$ (2) a fallacy

(3) a tautology (4) equivalent to $\sim p\to q$

Sol. **(3)**

p	Q	$p\to q$	$\sim p$	$\left(\sim p\to q\right)$	$\left(\sim p\to q\right)\to q$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	T

$(p \to q) \to [(\sim p \to q) \to q]$
T
T
T
T

48. The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the y-axis passes through the point:

(1) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Sol. **(4)**

Put $x = 0, y = 1$

In $y'(x-2)(x-3) + y(x-3) + y(x-2) = 1$

$\Rightarrow y' = 1$

Hence normal is $x + y = 1$

49. For any three positive real numbers a,b and c,

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

Then:

(1) a,b and c are in A.P. (2) a,b and c are in G.P.
(2) b,c and a are in G.P. (4) b,c and a are in A.P.

Sol. **(4)**

$225a^2 + 9b^2 + 25c^2 - 45ab - 15bc - 75ac = 0$

$(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2 = 0$

$15a = 3b = 5c$ only

$a = \frac{k}{15}, b = \frac{k}{3}, c = \frac{k}{5}$

50. If the image of the point $P(1, -2, 3)$ in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to :

(1) $\sqrt{42}$ (2) $6\sqrt{5}$ (3) $3\sqrt{5}$ (4) $2\sqrt{42}$

Sol. **(4)**

$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$

Image $Q(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$

Mid point by PQ $\left(\frac{\lambda + 2}{2}, \frac{4\lambda - 4}{2}, \frac{5\lambda + 6}{2}\right)$

Lie on $2x + 3y - 4z + 22 = 0$

$(\lambda + 2) + 3(2\lambda - 2) - 2(5\lambda + 6) = -22$

$-3\lambda = -22 - 2 + 6 + 12$

$\lambda = 2$

$P(1, -2, 3)$ $Q(3, 6, 13)$

$PQ = \sqrt{4 + 64 + 100} = 2\sqrt{42}$

51. If $5\left(\tan^2 x-\cos^2 x\right)=2\cos 2x+9$, then the value of $\cos 4x$ is :

(1) $\frac{2}{9}$ (2) $-\frac{7}{9}$ (3) $-\frac{3}{5}$ (4) $\frac{1}{3}$

Sol. **(2)**

$\cos^2 x = t$

$5\tan^2 x - 5\cos^2 x = 2\left(2\cos^2 x - 1\right)+9$

$\frac{5\left(1-t\right)}{t}-5t = 2\left(2t-1\right)+9$

$5-5t-5t^2 = \left(4t+7\right)t$

$9t^2+12t-5=0$

$9t^2+15t-3t-5=0$

$3t\left(3t+5\right)-1\left(3t+5\right)=0$

$t=\frac{1}{3},-\frac{5}{3}$

$\cos^2 x=\frac{1}{3}$

$\cos 2x = 2\cos^2 x-1$

$=\frac{2}{3}-1=-\frac{1}{3}$

$\cos 4x = 2\cos^2 2x-1$

$=2\left(-\frac{1}{3}\right)^2-1$

$=\frac{2}{9}-1=-\frac{7}{9}$

52. Let $a, b, c, \in \mathbf{R}$. If $f\left(x\right)=ax^2+bx+c$ is such that $a+b+c+=3$ and $f\left(x+y\right)=f\left(x\right)+f\left(y\right)+xy, \forall\, x, y\in \mathbf{R}$, then $\sum_{n=1}^{10} f\left(n\right)$ is equal to :

(1) 190 (2) 255 (3) 330 (4) 165

Sol. **(3)**

$f'\left(x+y\right)=f'\left(x\right)+y$

Put $x=0$, $f'\left(y\right)=f'\left(0\right)+y$ [Clearly $f\left(0\right)=0$]

$f'\left(y\right)=y+k$

$f\left(y\right)=\frac{y^2}{2}+ky+c$

$\therefore f\left(0\right)=0 \quad \therefore c=0$

$f\left(y\right)=\frac{y^2}{2}+ky$

$f\left(1\right)=\frac{1}{2}+k=3$

$k=\frac{5}{2}$

$\therefore f(x)=\frac{x^2}{2}+\frac{5}{2}x$

$\sum_{n=1}^{10} f(n)=330$

53. The distance of the point (1, 3, –7) from the plane passing through the point $(1,-1,-1)$, having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is :

(1) $\frac{5}{\sqrt{83}}$ (2) $\frac{10}{\sqrt{74}}$ (3) $\frac{20}{\sqrt{74}}$ (4) $\frac{10}{\sqrt{83}}$

Sol. **(4)**

Equation of plane

$$\begin{vmatrix} x-1 & y+1 & z+1 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}=0$$

Distance from $(1,3,-7)$ is $=\frac{10}{\sqrt{83}}$

54. If S is the set of distinct values of 'b' for which is following system of linear equations

$x+y+z=1$
$x+ay+z=1$
$ax+by+z=0$

has no solution, then S is :

(1) a finite set containing two or more elements
(2) a singleton
(3) an empty set
(4) an infinite set

Sol. **(2)**

$$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix}=0$$

By solving $a=1 \text{and} b\in R$

Then the planes are

$x+y+z=1$
$x+y+z=1$
$x+by+z=0$

If $b\neq 1$, infinite solution

If $b=1$, no solution

55. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$ then $\operatorname{adj}\left(3A^2+12A\right)$ is equal to :

(1) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$ (2) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$

(3) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$ (4) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$

Sol. **(4)**

$$A^2=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}=\begin{bmatrix}16 & -9\\ -12 & 13\end{bmatrix}$$

$$\therefore 3A^2+12A=\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$$

$$\Rightarrow \operatorname{Adj}\left(3A^2+12A\right)=\begin{pmatrix}51 & 63\\ 84 & 72\end{pmatrix}$$

56. A hyperbola passes through the point $P\left(\sqrt{2},\sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point :

(1) $\left(\sqrt{3},\sqrt{2}\right)$ (2) $\left(-\sqrt{2},-\sqrt{3}\right)$ (3) $\left(3\sqrt{2},2\sqrt{3}\right)$ (4) $\left(2\sqrt{2},3\sqrt{3}\right)$

Sol. **(4)**

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{2}{a^2}=\frac{3}{b^2}+1$$

$$a^2+b^2=4$$

$$\Rightarrow a^2=4-b^2$$

$$2b^2=3\left(4-b^2\right)+b^2\left(4-b^2\right)$$

$$\therefore b^2=3 \text{ and } a^2=1$$

So equation is $\frac{x^2}{1}-\frac{y^2}{3}=1$

Hence point is $\left(2\sqrt{2},3\sqrt{3}\right)$

57. Let k be an integer such that the triangle with vertices $(k,-3k),(5,k)$ and $(-k,2)$ has area 28 sq. units. Then the orthocenter of this triangle is at the point:

(1) $\left(1,-\frac{3}{4}\right)$ (2) $\left(2,\frac{1}{2}\right)$ (3) $\left(2,-\frac{1}{2}\right)$ (4) $\left(1,\frac{3}{4}\right)$

Sol. **(2)**

Area $=\frac{1}{2}\left(5k^2+13k+10\right)=28$

$K=2$

So vertices are $(2,-6)(5,2)(-2,2)$

Orthocenter is $\left(2,\frac{1}{2}\right)$

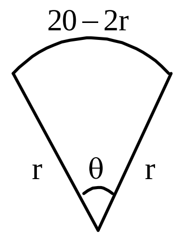
58. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :

(1) 25 (2) 30 (3) 12.5 (4) 10

Sol. **(1)**

Length of arc $= 20 - 2r = r\theta$

$$\Rightarrow \theta = 2\frac{(10-r)}{r}$$



Clearly $A = 10r - r^2$

Or $A = 25 - (r-5)^2$

So max value is 25 when $r = 5$

59. The function $f : \mathbf{R} \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is :

(1) surjective but not injective (2) neither injective nor surjective

(3) invertible (4) injective but not surjective

Sol. **(1)**

$f.R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$

Consider $f(x) = \frac{x}{1+x^2}$

To find range let $x = \tan(\theta/2)$

$f(x)$ is same as $\frac{\sin\theta}{2}$

$\Rightarrow \text{range}[-1/2, 1/2]$

For injection $f'(x) = \frac{1-x^2}{(1+x^2)^2}$ clearly non-monotonic

Hence not injective

60. A man X has 7 friends, 4 of them are ladies and 3 are men. His wire Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :

(1) 469 (2) 484 (3) 485 (4) 468

Sol. **(3)**

HUSBAND WIFE

3M	3F	–	$\left({}^4C_3\right)^2 = 4^2$
2M1F	2F1M	–	$\left({}^4C_2 . {}^3C_1\right)^2 = 18^2$
1 M2 F	1F 2m	–	$\left({}^4C_1 . {}^3C_2\right)^2 = 12^2$
3F	3M	–	$\left({}^3C_3\right)^2 = 1$
			485

PHYSICS

61. An observers is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light $3\times10^{8}\,ms^{-1}$)

(1) 12.1 GHz (2) 17.3 GHz (3) 15.3 GHz (4) 10.1 GHz

Sol. (2)

$f_0 = f_s\sqrt{\frac{1-\beta}{1+\beta}}$, where $\beta = \frac{v_0}{c}$

$v_0 = -\frac{c}{2}$ ($\because$ moving towards source)

$\therefore\ f_0 = 10\,GH_z\sqrt{\frac{1-\left(-\frac{1}{2}\right)}{1+\left(-\frac{1}{2}\right)}} = 17.3\,GHz$

62. The following observations were taken for determining surface tension T of water by capillary method: diameter of capillary, $D = 1.25\times10^{-2}\,m$ rise of water, $h = 1.45\times10^{-2}\,m$

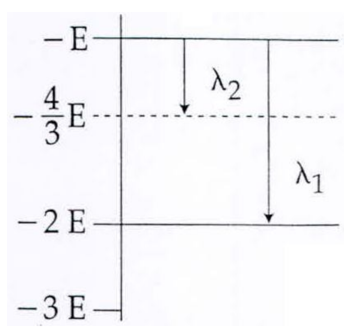
Using g = 9.80 m/s^2 and the simplified relation $T = \frac{rhg}{2}\times10^{3}\,N/m$, the possible error in surface tension is closest to:

(1) 1.5 % (2) 2.4 % (3) 10% (4) 0.15%

Sol. (1)

$$\%\,errors = \frac{0.01}{1.25}\times100 + \frac{0.01}{1.45} + \frac{0.01}{9.8}\times100$$
$$= 1.5\%$$

63. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by:



(1) $r = \frac{2}{3}$ (2) $r = \frac{3}{4}$ (3) $r = \frac{1}{3}$ (4) $r = \frac{4}{3}$

Sol. (3)

$E = \frac{hc}{\lambda_1}$ $\qquad 3 = \frac{\lambda_2}{\lambda_1}$

$\frac{E}{3} = \frac{hc}{\lambda_2}$ $\qquad$ or $\frac{\lambda_1}{\lambda_2} = 1/3$

64. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0{}^2$, the value of k will be:

(1) $10^{-3}\,kg\,s^{-1}$ (2) $10^{-4}\,kg\,m^{-1}$ (3) $10^{-1}\,kg\,m^{-1}\,s^{-1}$ (4) $10^{-3}\,kg\,m^{-1}$

Sol. (2)

$$\frac{dv}{dt} = -\frac{k}{m}v^2$$

$$\int_{10}^{5}\frac{1}{v^2} = -\frac{k}{10^{-2}}\int_{0}^{10}dt$$

$$\frac{1}{5} - \frac{1}{10} = -\frac{k}{10^{-2}}(10)$$

$$k = \frac{10^{-2}}{100} = 10^{-4}\,kg/m$$

65. C*P* and C*V* are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is:

(1) a = b (2) a = 14 b (3) a = 28 b (4) $a = \frac{1}{14}b$

Sol. (2)
For Hydrogen

$$C_p - C_v = a\frac{J\,K^{-1}}{2gm}$$

$$C_p - C_v = b\frac{J\,K^{-1}}{2gm}$$

$$C_p - c_v = b\frac{J\,k^{-1}}{28gm}$$

$$\frac{a}{2} = \frac{b}{28}$$

Or $14a = b$

$$a = \frac{b}{14}$$

66. The moment of inertia of a uniform cylinder of length *l* and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum?

(1) $\frac{\sqrt{3}}{2}$ (2) 1 (3) $\frac{3}{\sqrt{2}}$ (4) $\sqrt{\frac{3}{2}}$

Sol. (4)

$$I = \frac{mr^2}{4} + \frac{ml^2}{12}$$

$$= \frac{m}{4}\left(r^2 + \frac{l^2}{3}\right), \text{ as } \pi r^2 l\rho = m$$

$= \frac{m}{4}\left(\frac{m}{\pi\rho l} + \frac{l^2}{3}\right)$

For minimum I , $\frac{dI}{dl} = 0$

$-\frac{m}{\pi\rho l^2} + \frac{2l}{3} = 0$

$\Rightarrow \frac{l}{r} = \sqrt{\frac{3}{2}}$

67. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At some time t, the ratio of the number of B to that of A is 0.3. Then, t is given by:

(1) $t = T\frac{\log 1.3}{\log 2}$ (2) $t = T\log(1.3)$ (3) $t = \frac{T}{\log(1.3)}$ (4) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$

Sol. **(1)**

$N = N_0 e^{-\lambda t}$

$\frac{N_0}{1.3} = N_0 e^{-\lambda t}$

$\lambda t = \ln(1.3)$

$t = \frac{\ln(1.3)}{\ln 2}T$

$\lambda = \frac{hn2}{\lambda}$

68. Which of the following statements is false?

(1) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed

(2) A rheostat can be used as a potential divider

(3) Krichhoff's second law represents energy conservation.

(4) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

Sol. **(1)**

Theoretical

69. A capacitance of $2\mu F$ is required in an electrical circuit across a potential difference of 1.0 kV. A large number of $1\mu F$ capacitance are available which can withstand a potential difference of not more than 300 V

The minimum number of capacitors required to achieve this is

(1) 16 (2) 24 (3) 32 (4) 2

Sol. **(4)**

Let there be P branches in parallel and every branch consists of 'S' capacitors in series.

$\because C_{eq} = 2\mu f, \quad \therefore \left(P \times \frac{1\mu F}{S}\right) = 2\mu F$

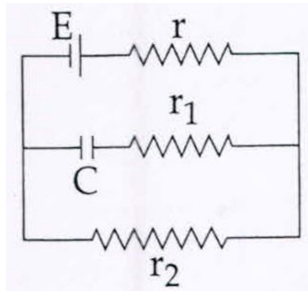
$\Rightarrow$ P = 2S … (i)

$\because$ a capacitor cannot withstand more than 300 V, So $\frac{1000}{S} \le 300$

$\Rightarrow S \geq \frac{10}{3} \qquad \Rightarrow S \geq 4 \qquad \qquad \ldots$ (ii)

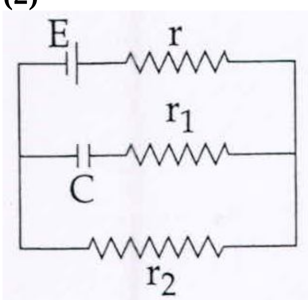
No of capacitors = $P \times S \geq 32$

70. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be:



(1) $CE\frac{r_1}{(r_2+r)}$ (2) $CE\frac{r_2}{(r+r_2)}$ (3) $CE\frac{r_1}{(r_1+r)}$ (4) CE

Sol. **(2)**



At steady state

$i_r = ir_2 = \frac{\varepsilon}{r+r_2}$

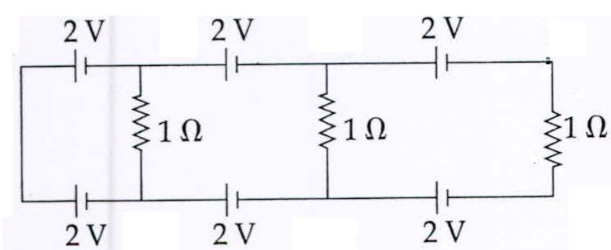
$i_{r},= 0$

$V_C = V_{r_2} = i_{r_2} \times r_2$

$= \frac{\varepsilon r_2}{r+r_2}$

$\therefore Q = CV_C = \frac{C\varepsilon r_2}{r+r_2}$

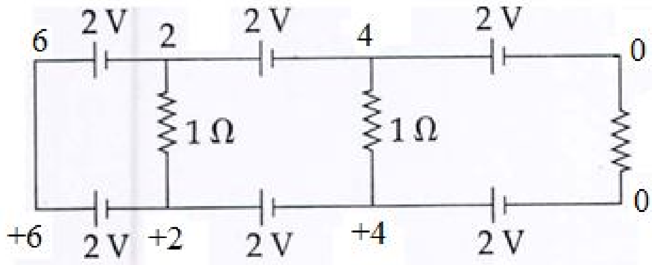
71.



In the above circuit the current in each resistance is

(1) 0.25 A (2) 0.5 A (3) 0 A (4) 1 A

Sol. **(3)**



72. In amplitude modulation, sinusoidal carrier frequency used in denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is **not** contained in the modulated wave ?

(1) ω_c (2) $\omega_m + \omega_c$ (3) $\omega_c - \omega_m$ (4) ω_m

Sol. **(4)**

Refer Theory

73. In a common emitter amplifier circuit using an n–p–n transistor, the phase difference between the input and the output voltage will be

(1) 90^0 (2) 135^0 (3) 180^0 (4) 45^0

Sol. **(3)**

Refer Theory

74. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75^0C. T is given by:

(Given: room temperature = 30^0C, specific heat of copper = 0.1 cal/gm^{0}C)

(1) 885^0C (2) 1250^0C (3) 825^0C (4) 800^0C

Sol. **(1)**

Heat released by Cu ball = Heat absorbed by water + Calorimeter

$$M_{Cu}S_{Cu}\Delta T = m_w S_w \Delta T + m_{cal} S_w \Delta T$$

$$100 \times 0.1 \times (T - 75) = 170 \times 1 \times (75 - 30) + 100 \times 0.1 \times (75 - 30)$$

$$T = 885^0 C$$

75. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is:

(1) 7.8 mm (2) 9.75 mm (3) 15.6 mm (4) 1.56 mm

Sol. **(1)**

$$\frac{n_1 D\lambda_1}{d} = \frac{n_2 D\lambda_2}{d}$$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{4}{5}$$

$$y = \frac{n_1 D\lambda_1}{d} = \frac{4 \times 1.5 \times 65 \times 10^{-8}}{5 \times 10^{-4}} = 7.8\,\text{mm}$$

76. An electric dipole has a fixed dipole moment $\vec{P}$, which makes an angle θ with respect to x – axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = {}_\tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

(1) 45^0 (2) 60^0 (3) 90^0 (4) 30^0

Sol. **(2)**

$\tau = \vec{P}\times\vec{E}$

$\because \vec{E}_1$ & $\vec{E}_2$ don't have $\hat{k}$ component and $\vec{\tau}_1$ & $\vec{\tau}_2$ only have $\hat{k}$ component so $\vec{P}$ will be $\hat{i}-\hat{j}$ plane.

Let $\vec{P} = P_x\hat{i} + P_y\hat{j}$

$\because \vec{\tau}_1 = \tau\hat{k} = \left(P_x\hat{i} + P_y\hat{j}\right)\times\left(E\hat{i}\right)$

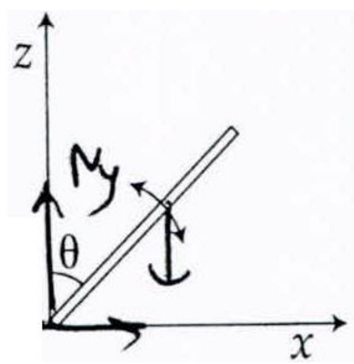
$\Rightarrow \tau = -P_yE \quad \ldots (1)$

$\because \overrightarrow{\tau_2} = -\tau\hat{k} = \left(P_x\hat{i} + P_y\hat{j}\right)\times\left(\sqrt{3}E_1\hat{j}\right)$

$\Rightarrow -\tau = \sqrt{3}P_xE$

$\therefore \theta = 60^0$

77. A slender uniform rod of mass M and length *l* is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



(1) $\frac{2g}{3l}\sin\theta$ (2) $\frac{3g}{3l}\cos\theta$ (3) $\frac{2g}{3l}\cos\theta$ (4) $\frac{3g}{2l}\sin\theta$

Sol. **(4)**

$\tau = I\alpha$

$\alpha = \frac{3g}{2\ell}\sin\theta$

78. An external pressure P is applied on a cube at 0^0C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

(1) $\frac{P}{\alpha K}$ (2) $\frac{3\alpha}{PK}$ (3) $3PK\alpha$ (4) $\frac{P}{3\alpha K}$

Sol. **(4)**

$B = \frac{P}{\frac{\Delta V}{V}}$ or $\Delta V = \frac{PV}{B}$

Also,

$\Delta V = V.\gamma\Delta T = V.3\alpha\Delta T$

$V.3\alpha\Delta T = \frac{pV}{B}$

Or $\Delta T = \frac{P}{3B\alpha}$

79. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is:

(1) virtual and at a distance of 40 cm from convergent lens
(2) real and at a distance of 40 cm from the divergent lens
(3) real and at a distance of 6 cm from the convergent lens
(4) real and at a distance of 40 cm from convergent lens

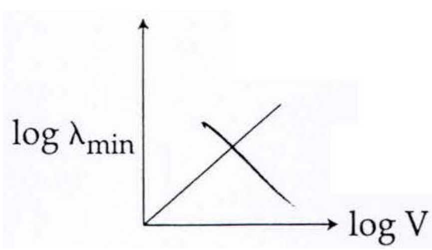
Sol. **(1)**

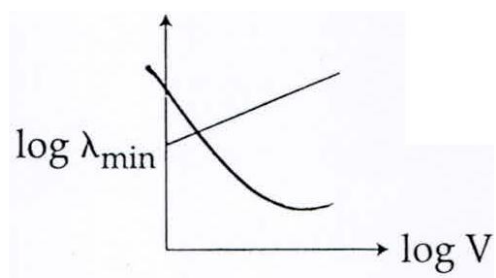
For $L_2 : \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

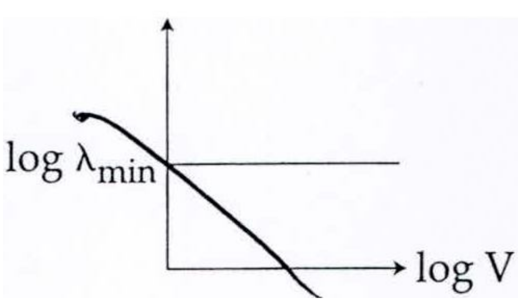
$\frac{1}{+20} = \frac{1}{v_2} - \frac{1}{-40}$

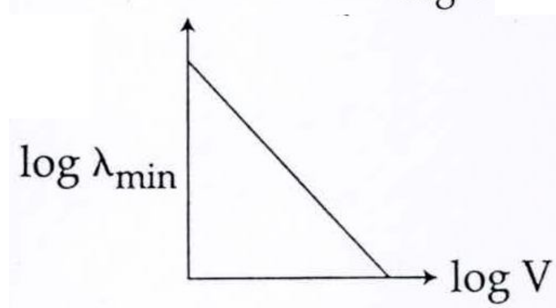
$v_2 = +40\,\text{cm}$

80. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X – rays. It produces target to produce X – rays. It produces continuous as well as characteristic X – rays. If λ_{min} is the smallest possible wavelength of X – ray in the spectrum, the variation of $\log\lambda_{min}$ with log Y is correctly represented in

(1)


(2)


(3)


(4)


Sol. **(4)**

$\lambda_c = \frac{hc}{ev}$

$\lambda_c V = \frac{hc}{e} = \text{constant}(\alpha)$

$\ln\lambda_c + \ln V = \ln(\alpha)$

$\ln\lambda_c = -\ln V + \text{cosntant}$

81. The temperature of an open room of volume 30 m^3 increases from 17^0C to 27^0C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa.

(1) 1.38×10^{23} (2) 2.5×10^{25} (3) -2.5×10^{25} (4) -1.61×10^{23}

Sol. **(3)**

$$n_f - n_0 = \frac{pv}{R}\left[\frac{1}{T_f}-\frac{1}{T_0}\right]$$

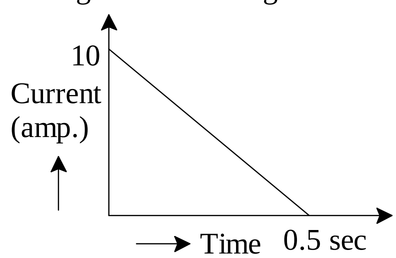
$$=\frac{1\times10^5 pa\times30m^3}{8.314J\,mol^{-1}x^{-1}}\left[\frac{1}{300-\frac{1}{290}}\right]$$

$$=-41.47\,mol$$

$$=-41.87\times6.02J\times10^{23}$$

$$=-2.5\times10^{25}$$

82. In a coil of resistance 100Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :



(1) 225 Wb (2) 250 Wb (3) 275 Wb (4) 200 Wb

Sol. **(2)**

$$2.5=\frac{d\phi}{R} \qquad 2.5\times100$$

83. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is :

(1) $2.045\times10^3\Omega$ (2) $2.535\times10^3\Omega$ (3) $4.005\times10^3\Omega$ (4) $1.985\times10^3\Omega$

Sol. **(4)**

$$V = ig(R+G)$$

$$\frac{10}{5\times10^{-3}}-G=R$$

$$\frac{1}{2}\times10\times0.5=2.5$$

2000 – 15 = R

1985

84. A time dependent force F = 6t acts on a particle of mass 1 kg. if the particle starts from set, the work done by the force during the first 1 sec. will be :

(1) 22 J (2) 9 J (3) 18 J (4) 4.5 J

Sol. **(4)**

$$F = 6t$$

$$m\frac{dv}{dt}=6t$$

$$\int_0^v dv = \int_0^t 6tdt$$
$$v = 3t^2$$
Work done $= \Delta k$
$$= \frac{1}{2}m v_f^2 - \frac{1}{2}mv_i^{\,2}$$
$$= \frac{1}{2}(1)(3)^2 - 0$$
$$= 4.5J$$

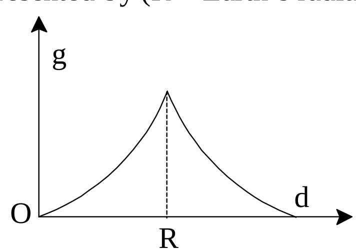
85. A magnetic needle of magnetic moment $6.7\times10^{-2}\,Am^2$ and moment of inertial 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is :

(1) 8.89 s (2) 6.89 s (3) 8.76 s (4) 6.65 s

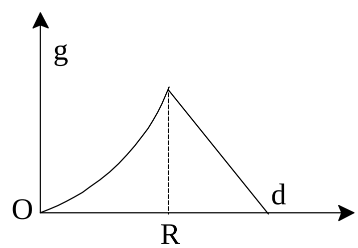
Sol. **(4)**

$$2\pi.\sqrt{\frac{I}{MB}} = 2\pi\sqrt{\frac{7.5\times10^{-6}\,10^{-2}}{6.7\times10^{-2}\times0.01}}\times10$$

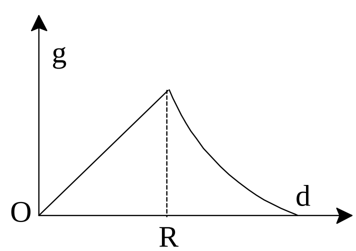
86. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :



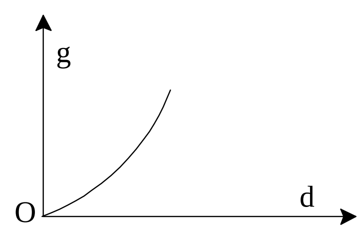
(1)



(2)



(3)

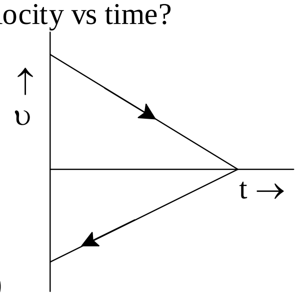


(4)

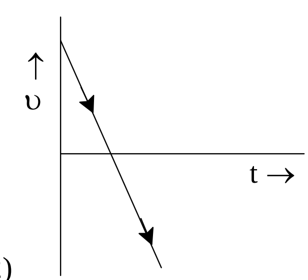
Sol. **(3)**

Theoretical

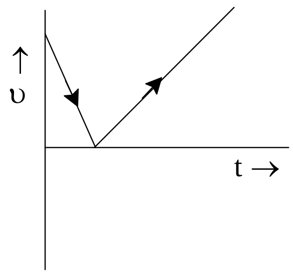
87. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

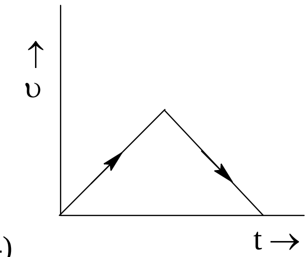


(1)



(2)

(3) 

(4) 

Sol. (2)

$\uparrow^{u}$ Initial velocity is +ve

$$\frac{dV}{dt} = -g$$

$$\int_U^V dV = \int_0^t g\,dt$$

$$V - U = -gt$$

$V = U - gt$ (straight line with negative slope)

88. A particle A of mass m and initial velocity υ collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelength λ_A to λ_B after the collision is :

(1) $\frac{\lambda_A}{\lambda_B} = 2$ (2) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (3) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (4) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

Sol. (1)

$$mv = mv_1 + \frac{m}{2}v_2$$

$$v = v_1 + \frac{v_2}{2} \quad \ldots (1)$$

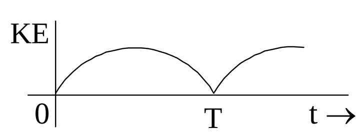
$$v - 0 = v_2 - v_1 \quad \ldots (2)$$

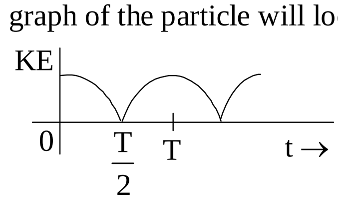
On solving

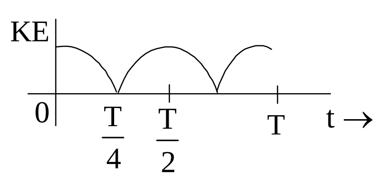
$$v_1 = v/3, v_2 = 4v/3$$

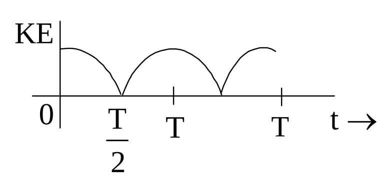
$$\frac{\lambda_1}{2} = \frac{h/m_1v_1}{h/m_2v_2} = 2$$

89. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like:

(1) 

(2) 

(3) 

(4) 

Sol. (3)

90. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that this density remains same, the stress in the leg will change by a factor of :

(1) $\frac{1}{9}$ (2) 81 (3) $\frac{1}{81}$ (4) 9

Sol. **(4)**

$\text{Stress} = \frac{Mg}{A}$

$= \frac{\rho v g}{A}$

$= \frac{\rho q^3 v g}{q^2 A}$

$= q$